THE PROCEEDINGS OF THE PHYSICAL SOCIETY

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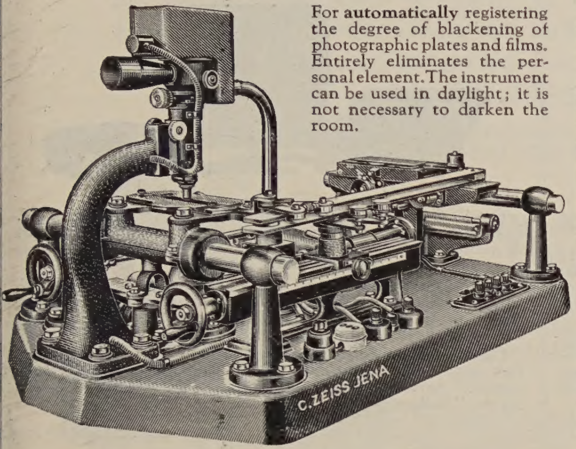

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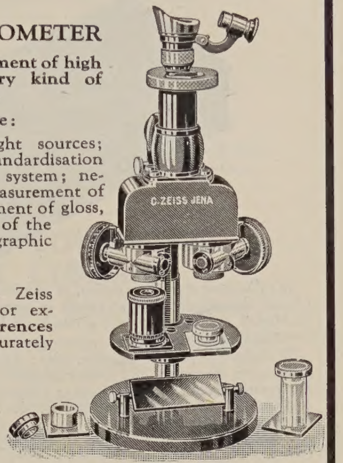

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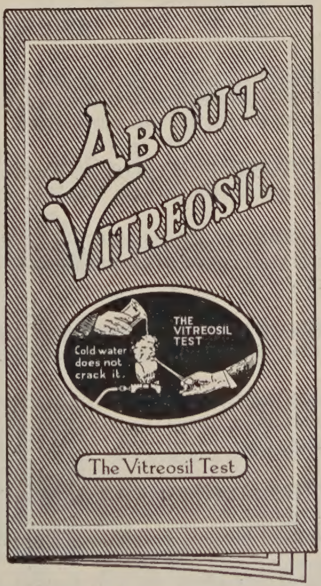

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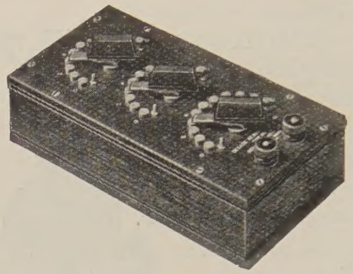

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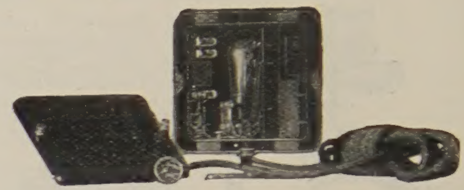

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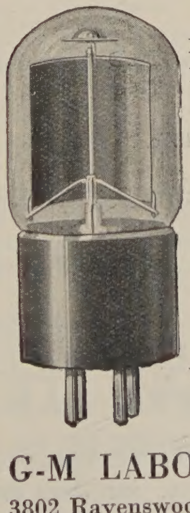

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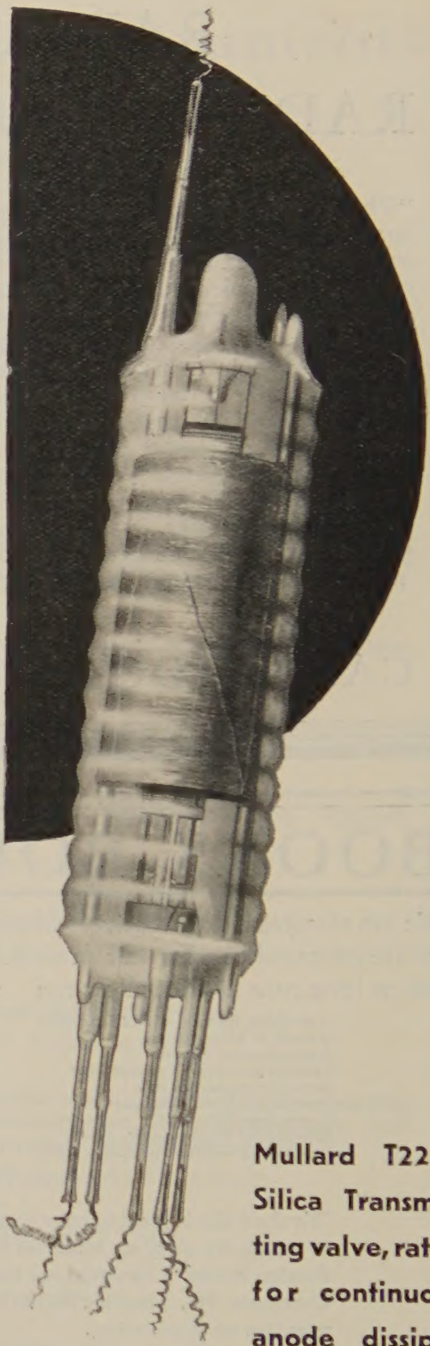

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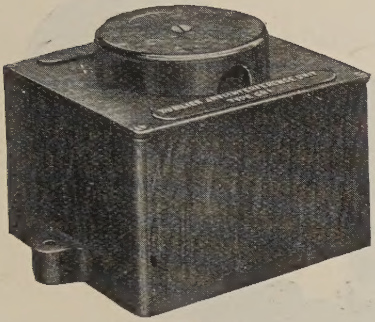

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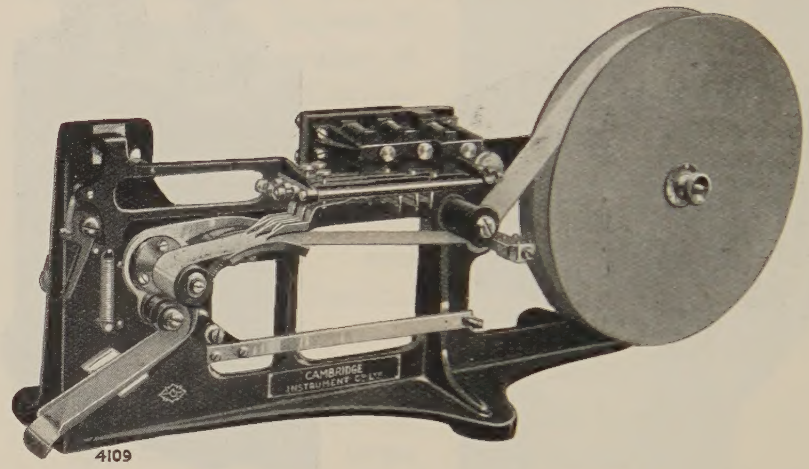

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PHYSICAL AND OPTICAL SOCIETIES

TWENTY-FIRST ANNUAL EXHIBITION OF APPARATUS

Opening address delivered by the President

SIR A. S. EDDINGTON, F.R.S.

THE present occasion marks the coming of age of this Exhibition, which has become an outstanding annual event in the scientific world, for it is the 21st of the series. It is therefore appropriate to recall the history of this informal congress of the makers and the users of scientific instruments.

It started on December 15, 1905, when the Physical Society held an evening meeting at the Royal College of Science at which certain instrument makers by invitation exhibited their latest productions. Tickets were issued to the members of the Optical Society and of the Institution of Electrical Engineers also. Seventeen firms, most of which are represented on the present occasion, then exhibited. The attendance was about 240—greater than had been anticipated.

The usefulness of and interest in the Exhibition increased year by year. In 1909 the attraction of discourses by leading physicists was added. The first two of these discourses were given by Prof. C. V. Boys and the late Prof. Sylvanus Thompson. This year we are to hear Mr E. Lancaster-Jones and Sir Gilbert Walker.

There was a suspension during the war. On the resumption in 1920 the number of exhibiting firms had risen to 72. Perhaps I should say that this was the number of voluntary exhibitors, because on that occasion there were also a large number of captured German instruments on view.

At this stage the Optical Society joined in. The Exhibition had hitherto been managed by the Honorary Secretary of the Physical Society personally at great sacrifice. It was now entrusted to a Joint Committee of the Physical and Optical Societies, the expense being shared between the two bodies; and I must not leave out a third party whose action makes the holding of this Exhibition possible, namely the Governors of the Imperial College of Science, who have throughout this long period recognized the value of the Exhibition to workers in Science and have freely placed accommodation at our disposal. It is not only from a financial standpoint that we appreciate this. This is not a trade exhibition in the ordinary sense; and the holding of it within the walls of a great institution of learning and research creates an atmosphere which we should find it hard to imitate in any public exhibition hall. By distribution of the exhibits among the college laboratories use can be made of the facilities for demonstrating the apparatus under working conditions.

In 1926 three new sections were added representing (*a*) typical results of recent research, (*b*) effective but little-known lecture experiments, and (*c*) apparatus of historical interest and repetitions of famous experiments.

The research section is of especial interest and value. Reading the catalogue I cannot but be struck with the conspicuous part taken in it by the research laboratories of the great commercial firms. That is one of the interesting developments of the present day. The field of pure research is no longer a matter solely for university laboratories.

In 1926 it was decided, in view of the general public interest in scientific developments, that the duration of the Exhibition should be increased by one day so that the general public might be freely admitted without detriment to the character of the exhibition as a meeting-place for the makers and users of scientific instruments. This year the public day is Thursday, January 8.

Last year another innovation was introduced, namely a competition for craftsmanship and draughtsmanship, the work of learners in the exhibiting firms. Judges appointed by the Councils of the two Societies viewed the fifty-three exhibits certified as the unaided work of individual apprentices, and awarded prizes subscribed for by the employers. No visitor could fail to be impressed by the high standard attained, and it is believed that the new section will have beneficial results in encouraging and raising the standard of British workmanship and skill. The exhibits in this section on the present occasion will be on view in the Science Museum for a few weeks after the close of the exhibition, by the courtesy of the Director.

An Exhibition like this brings home to us two things—the debt of the scientific worker to the instrument-maker and the debt of the instrument-maker to the scientific worker. I do not know which to place first. I do not think one can be placed before the other; for the instrument-maker provides the resources of the scientific worker, and the scientific worker provides the resources of the instrument-maker. It is of great interest to those of us who are interested in the purely scientific side to see how discovery and progress has been turned to account in these applications, and to see the practical working of scientific method. My instrument-maker friends, I know, like to try to shock me by professing to be unscientific. "We don't really calculate out these curves and dimensions," they say "we go on by trial and error until it comes right." That does not shock me in the least, for "trial and error" can be as scientific as any other method. It is indeed one of the most efficient devices for the harder problems not only in pure physics but in mathematics; and I know well the skill and systematic planning that are needed to make it an instrument of progress. Nor do I think less of the instrument-designer because he sees by trained instinct what we can only worry out by laborious formulae. I look on him as the master chess-player who grasps the strong and the weak positions of the game intuitively, whilst we inexperienced players have to analyse the situation by moving the pieces one by one.

But there is another less direct debt which the instrument-maker owes to the scientific worker—the stimulus to satisfy new requirements. How often does it not happen that an instrument is designed to meet some new development of scientific research, perhaps with no thought that more than one instrument of the type will ever be needed; a few years later the same instrument is being turned out in hundreds as an indispensable adjunct to commercial operations.

I suppose that the earliest scientific instrument of any high degree of elaboration and delicacy of workmanship belonged to my own science, namely the Astrolabe. We can read in Chaucer of the astrolabe which he obtained for "Litel Lowis my sone, having perceived wel by certeyne evidences thyn abilite to lerne sciencez touchinge noumbres and proporciouns." I cannot forbear quoting a few lines, since it is in a style somewhat different from most of our handbooks on astronomical instruments.

> Thyn Astrolabie hath a ring to putten on the thoumbe of thy right hand in taking the heighte of thinges. And tak keep, for from hennes-forthward I wol clepe the heighte of anything that is taken by thy rewle, the altitude, withoute mo wordes....The est side of thyn Astrolabie is cleped the right side, and the west side is cleped the left side. Forget nat this, litel Lowis.

Ever since the days of the astrolabe, astronomy has been one of the most pressing of the sciences in its demands on the instrument-maker. It is well-known how the telescope was invented by a Dutch spectacle-maker. Testing his lenses by looking through them at a church spire he happened to place two, one behind the other, and saw a greatly enlarged image of the spire. The telescope has had a varied history. At one time before the invention of achromatic lenses, in order to get rid of the colour difficulty, telescopes ran to enormous and unwieldy lengths. Telescopes 300 and 600 feet long were made. The longest telescope of which we have any record of actual use in observation was 212 feet long; it was used in 1722 by Bradley for measurements of the planet Venus. Compared with this, even the 100-inch reflector at Mount Wilson is a handy little instrument. The old Statutes of my own Professorial Chair, which was created about this time, lay it down that the Professor should observe with a telescope not less than 16 feet long—which I suppose was a very modest demand for that period. I am glad to say that, notwithstanding the greater compactness of modern instruments, I fulfil this requirement—but without much margin to spare.

The discovery of achromatic object-glasses in 1733 and the development of reflecting telescopes have changed all that. Achromatism in particular illustrates what I have said as to the way in which an instrumental development produced for some purely scientific purpose is later employed in all kinds of practical applications. I think that at that time the telescope was almost wholly an instrument for astronomical purposes; indeed its unwieldiness almost precluded any commoner application. And it was the urgent need for the improvement of astronomical observation which called forth the invention of colour-free lenses. Nowadays there is hardly an instrument of any complexity which does not employ an achromatic lens in some part of its system.

As an astronomer I am filled with the deepest gratitude to the practical instrument-maker. Gratitude has been defined as "an expectation of favours to come," and I will not reject that definition in my own case. For a long time to come we shall be asking for more, making those impossible demands which sooner or later the instrument-maker and his research staff bring within the bounds of possibility. At present great hopes are centred on the projected 200-inch reflector which the Carnegie Institution has decided upon, doubling the aperture of the present largest telescope—the 100-inch at Mount Wilson. That will undoubtedly carry us a long way further into the mysteries of the universe. Yet it is interesting to reflect that just the same results might come about by one of those simple, almost accidental, discoveries in a research laboratory. Suppose that whether by further insight into the theory of photochemical action, or by systematic application of the method of "trial and error" which, as I have said, is also a truly scientific procedure, we can produce a photographic plate four times as sensitive as those at present in astronomical use; that would effectively turn the present 100-inch into a 200-inch, and at one stroke bring within reach all that is expected from the great new telescope. In fact for many purposes, such as the exploration of the nebulae, it would be more effective. And not only would such a discovery make the 100-inch equivalent to a 200-inch, but virtually it would double the aperture of every photographic telescope throughout the world.

Our acknowledgment of ever-increasing debt to, and dependence on, the instrument-maker is not without a touch of regret for the good old days. Astronomers, it is true, have long been dependent on highly complex and expensive instruments, which once acquired have to be used to the utmost. For most observatories the question is not so much what kind of astronomical work offers the most promising prospects at the moment as what kind of astronomical work their particular equipment is best suited for. But until lately the physicist was free from that kind of limitation, and designed his apparatus to suit his work rather than designed his work to suit his apparatus. The days are past when one could put together in the workshop some "tin-tacks and sealing-wax," and then set to work and discover something. Times change and we must move with the times. And here we have on view the latest appliances with which we can equip ourselves for further and further advance, to wrest their secrets from the electron, the atom and the star.

Now Ladies and Gentlemen, we are standing before the entrance of a magic cave containing wonders worthy of the Arabian Nights. You will not find therein sacks of gold and diamonds and rubies, for I think that neither the maker nor the user of scientific instruments has very much to display of that kind of wealth. Nevertheless, it is a cave of treasure—the treasure that is evolved by the cunning of the brain and the treasure that is evolved by the cunning of the hand. In this complex civilization of ours, wherever we turn, we meet with the applications of science; and here you will find gathered together the nucleus of them all—the linking of pure science to technical industry. The eastern story was not far wrong, when it placed the treasure under the charge of the Slave of the Lamp, to be possessed by whoever should learn how to control him. You will find very much in evidence in our treasure-cave the Slave of the up-to-date Lamp—electricity. It remains for me to pronounce the words that shall throw open the doors of the cave. Open Sesame!

A POINT OF ANALOGY BETWEEN THE EQUATIONS OF THE QUANTUM THEORY AND MAXWELL'S EQUATIONS

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ABSTRACT. It is known that there is a close analogy, exhibited by the use of five-dimensional tensor notation, between the electromagnetic equations in free space and the first- and second-order equations of the quantum theory. In the present paper an analogy is traced between two tensors, one of which is related to the magnetic and electric moments of a doublet while the other represents the electric and magnetic field-strengths.

DARWIN*, WHITTAKER†, FRENKEL‡ and others have shown that there is a close analogy between the electromagnetic equations in free space and the first-order equations of the quantum theory. These equations were first developed by Dirac from considerations which have nothing to do with this analogy. The resulting second-order equations contain, in addition to those appearing in Schrödinger's equation, terms which are just those required to account for the duplexity phenomena of the atom.

The general form of the electromagnetic equations in four-dimensional space is:

$$\frac{1}{g^{\frac{1}{2}}}\frac{\partial}{\partial x^n}(g^{\frac{1}{2}}F^{mn}) = J^m \qquad \ldots\ldots(1),$$

$$\frac{1}{g^{\frac{1}{2}}}\frac{\partial}{\partial x^n}(g^{\frac{1}{2}}G^{mn}) = K^m \qquad \ldots\ldots(2).$$

Flint and Fisher§ have generalized a system of equations in five-dimensional space to take the place of (1). They are written in the following form:

$$\frac{1}{\gamma^{\frac{1}{2}}}\frac{\partial}{\partial x^\nu}(\gamma^{\frac{1}{2}}A^{\mu\nu}) - \frac{2\pi i}{h}\Pi_\nu A^{\mu\nu} = 0 \qquad \ldots\ldots(3),$$

Π_a
p_a
ϕ_a
$A^{\mu\nu}$

where J^m is put equal to zero and additional terms have been introduced which correspond to the metric adopted. $\Pi_a = (p_a - e\phi_a)$, where Π_a is a momentum introduced by W. Wilson|| in the study of the motion of the electron, p_a denotes the component of four-dimensional momentum and ϕ_a is the electromagnetic potential. $A^{\mu\nu}$ is an antisymmetric tensor whose metrical significance has not yet been found. The authors referred to have also developed second-order equations which are

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* *Proc. R. S.* A, **118**, 654 (1928).

† *Proc. R. S.* A, **121**, 543 (1928).

‡ *Einführung in die Wellenmechanik.*

§ *Proc. R. S.* A, **126**, 644 (1930).

|| *Proc. R. S.* A, **102**, 478 (1923).

identical with those of Frenkel and Whittaker. As an example of one of these we may write the following:

$$\pentagon\psi_0 - \frac{2\pi ie}{h}(H_1A^{23} + H_2A^{31} + H_3A^{12} - iE_1A^{14} - iE_2A^{24} - iE_3A^{34}) = 0 \qquad \ldots\ldots(4),$$

where $$\pentagon\psi_0 = \Sigma\frac{\partial^2\psi_0}{\partial x^2} - \frac{4\pi ie}{h}\phi_m\frac{\partial\psi_0}{\partial x^m} - \frac{4\pi^2}{h^2}(\Sigma e^2\phi_m{}^2 + m^2c^2)\psi_0 \qquad \ldots\ldots(5).$$ $\pentagon$

The foregoing considerations were based upon a close analogy with the propagation of light as represented in a four-dimensional continuum. The track of a light quantum in this continuum is a nul geodesic

$$ds^2 = 0.$$

The equations of Maxwell for empty space are those which we obtain by writing $J^m = 0$ and $K^m = 0$ in (1) and (2). The view of Flint and Fisher is that the nul geodesic $d\sigma^2 = 0$ for a five-dimensional continuum will give the track of the electron and that the equations in this space corresponding to (1) and (2) will give the first-order equations of the quantum theory. In the paper referred to above they develop this idea.

The object of the present paper is to bring to light another point in the analogy between this system of equations and those of Maxwell. Equation (4) can be written in the following vectorial form:

$$\pentagon\psi_0 - \frac{2\pi ie}{h}(H.a - iE.b) = 0 \qquad \ldots\ldots(6),$$

where $$a_1 = A^{23},\quad a_2 = A^{31},\quad a_3 = A^{12};$$ a, b

$$b_1 = A^{14},\quad b_2 = A^{24},\quad b_3 = A^{34}.$$

Writing $$a = \frac{4\pi im\psi_0}{he}M \quad\text{and}\quad b = -\frac{4\pi m\psi_0}{he}N$$ M, N

we can put equation (6) into the following form:

$$\pentagon\psi_0 + \frac{8\pi^2m}{h^2}(H.M + E.N)\psi_0 = 0 \qquad \ldots\ldots(7).$$

But Schrödinger's wave equation is

$$\nabla^2\psi_0 + \frac{8\pi^2m}{h^2}(W - U)\psi_0 = 0 \qquad \ldots\ldots(8),$$

which is a special case of the equation $\pentagon\psi_0 = 0$.

From (7) and (8) we get

$$\pentagon\psi_0 = \nabla^2\psi_0 + \frac{8\pi^2m}{h^2}(W - U)\psi_0 + \frac{8\pi^2m}{h^2}(H.M + E.N)\psi_0 \quad \ldots\ldots(9).$$

The last term of equation (9) represents an additional energy due to a doublet. From this we infer that M is the magnetic moment and N is the electric moment of the doublet. Our antisymmetric tensor $A^{\mu\nu}$ is therefore evidently related in some simple way to these two quantities.

The complete scheme for $A^{\mu\nu}$ is:

$$A^{\mu\nu} = \left|\begin{array}{ccccc} 0 & \frac{4\pi i m}{he} M_3\psi_0 & -\frac{4\pi i m}{he} M_2\psi_0 & -\frac{4\pi m}{he} N_1\psi_0 & \frac{4\pi m}{he} N_1\psi_0 \\ -\frac{4\pi i m}{he} M_3\psi_0 & 0 & \frac{4\pi i m}{he} M_1\psi_0 & -\frac{4\pi m}{he} N_2\psi_0 & \frac{4\pi m}{he} N_2\psi_0 \\ \frac{4\pi i m}{he} M_2\psi_0 & -\frac{4\pi i m}{he} M_1\psi_0 & 0 & -\frac{4\pi m}{he} N_3\psi_0 & \frac{4\pi m}{he} N_3\psi_0 \\ \frac{4\pi m}{he} N_1\psi_0 & \frac{4\pi m}{he} N_2\psi_0 & \frac{4\pi m}{he} N_3\psi_0 & 0 & i\psi_0 \\ -\frac{4\pi m}{he} N_1\psi_0 & \frac{4\pi m}{he} N_2\psi_0 & \frac{4\pi m}{he} N_3\psi_0 & -i\psi_0 & 0 \end{array}\right| \qquad \ldots\ldots(10),$$

or

$$A^{\mu\nu} \propto \left|\begin{array}{ccccc} 0 & M_3 & -M_2 & iN_1 & N_1 \\ -M_3 & 0 & M_1 & iN_2 & N_2 \\ M_2 & -M_1 & 0 & iN_3 & N_3 \\ -iN_1 & -iN_2 & -iN_3 & 0 & \frac{he}{4\pi m} \\ -N_1 & -N_2 & -N_3 & -\frac{he}{4\pi m} & 0 \end{array}\right| \qquad \ldots\ldots(11).$$

The complete scheme for F^{mn} is:

$$F^{mn} = \left|\begin{array}{cccc} 0 & -H_z & H_y & iE_x \\ H_z & 0 & -H_x & iE_y \\ -H_y & H_x & 0 & iE_z \\ -iE_x & -iE_y & -iE_z & 0 \end{array}\right| \qquad \ldots\ldots(12).$$

Comparing (11) with (12) we notice that $A^{\mu\nu}$ is a tensor in five-dimensional space whose components are related in a simple way to the magnetic and electric moments of the doublet, while F^{mn} is a tensor in four-dimensional space whose components are the electric and magnetic field-strengths.

SOURCES OF ILLUMINATION FOR ULTRA-VIOLET MICROSCOPY

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ABSTRACT. The paper describes experiences encountered in an attempt to find means for reducing the exposures hitherto necessary in ultra-violet microscopy. Amongst the subjects dealt with are quantitative measurements of the relative intrinsic brightness of spectrum lines given by various sources of radiation; methods of producing a monochromatic source suitable for this work; the steadiness of the illuminant; the effect on the definition of the image when a triple spectrum line is used as a source; the study of electrical conditions for the production of increased brightness of the spark, such as the effects of change in frequency, secondary potential, capacity, and energy input; and the production of a compact and inexpensive electrical unit for use with the ultra-violet microscope.

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§ 1. INTRODUCTORY

One of the difficulties in ultra-violet microscopy is the provision of a suitable source of radiation which has sufficient luminosity to enable photomicrographs to be taken with a short exposure. Up to the present, when the highest-power immersion monochromat objective is used, the duration of exposure for the transparent type of objects, such as those used in biological and allied work, has been of the order of 10 to 30 seconds, whilst in metallurgical work exposures of 3 and 4 minutes have been found necessary.

In biology, an exposure of the length of 10 seconds necessitates "fixing" of the object in order that Brownian and other movements may be prevented. Moreover such an exposure to ultra-violet radiation of the wave-length at present employed has in some cases a germicidal action on the specimen. For metallurgical objects 3 or 4 minutes' duration of exposure may under certain conditions—for example, instability of the microscope or a change of refractive index of the immersion fluid—prevent precisely focussed photographs from being obtained, and from an economical point of view a reduction in this exposure would seem advisable. In other branches of work also, in which the ultra-violet microscope could be used, the question of shortness of exposure is likely to be of importance, and therefore an attempt has been made to find means for reducing the length of exposure at present required for the taking of ultra-violet photomicrographs generally. An account of the work carried out forms the subject of this paper.

§ 2. THEORY

As the photographic density of the image on the plate is the ultimate criterion, the following relation is the one concerned in this work:

$$\text{Illumination of unit area of image} \propto B \times \left(\frac{\text{N.A.}}{m}\right)^2 \times t,$$

where B is the intrinsic brightness of source,
N.A. the numerical aperture of microscope objective,
m the magnification employed, and
t the transmission factor of the optical parts.

It is evident from this that when once the objective has been decided on and the numerical aperture therefore fixed, the only two variables in the formula are B and m. The latter quantity, however, must not fall below a limiting value if the grain of the plate is to resolve the image, and it is obviously better to use this minimum value where short exposures are desired, rather than to employ high "empty" magnification. This leaves us, therefore, with the increasing of the brightness of the source as the only hope for a reduction in exposure.

The conditions which govern an emission source suitable for this work are as follows: (1) the radiation given by the source must be strictly monochromatic, i.e. must comprise one spectrum line with no other intense lines near it; (2) the source should give maximum possible energy in this region; (3) for existing monochromats the wave-length of the line must be between 0·29μ and 0·25μ.

It is seen, at once, that these conditions place considerable restrictions on the choice of the most suitable source from those which are available. As regards condition (2), it is well known that the amount of energy in the spectra produced by the flame and the arc is greatest in the infra-red region, whilst in the solar spectrum the maximum lies in the visible green, whereas in the case of the spark spectrum the maximum energy is concentrated in the ultra-violet region. The following figures give an illustration of the relative energy distribution in the spark spectrum:

Infra-red	10
Visible	24
Ultra-violet	66

It will, then, be found advisable to employ the spark method for producing the illuminant, for it appears (as described later) that the intrinsic brightness of the spark is proportional to the amount of energy liberated across the spark gap.

Condition (1) enforces the use of some means whereby the desired line can be isolated. This result could be obtained either by dispersion through quartz prisms, by dispersion from a concave reflexion grating, or by use of the illuminant directly with a suitable filter of narrow transmission limits. Up to the present time, however, the first of these methods is the only one which has proved practicable, and therefore a monochromator, consisting of two 60° quartz prisms together with quartz collimator and telescope lens, has to be arranged suitably in conjunction with the source.

As yet it has been necessary to compute ultra-violet microscope objectives for one wave-length only and to use them strictly with this particular radiation; it is, however, permissible to use them with a wave-length which does not differ very widely from the original computed radiation, without introducing any serious aberration effects which might affect the definition of the image. For instance, a monochromat intended for use with $\lambda = 0{\cdot}275\,\mu$ can be used quite satisfactorily with the cadmium monochromatic line $\lambda = 0{\cdot}257\,\mu$. Consequently the experiments here described were confined to a limited range within which it was considered safe to use the objective, namely between $0{\cdot}29\,\mu$ and $0{\cdot}25\,\mu$.

§ 3. EXPERIMENTAL WORK

Attempts were first made to determine the relative brightness of different sources of radiation which might be used under the conditions laid down above. The apparatus for doing this is shown diagrammatically in figure 1. A cadmium photo-electric cell* sensitive to radiations between $0{\cdot}2\,\mu$ and $0{\cdot}3\,\mu$ was employed with the quartz monochromator M arranged in front of it. The quartz lens L was used to form an image of the iris diaphragm I on the mouth of the cell; by this means fluctuation of the source, a difficulty which was at first experienced when the image of the source was focussed directly on the cell window, was eliminated. All parts of the apparatus were securely positioned and provision was made for ensuring also precise location and width (namely 3 mm.) of the spark gap.

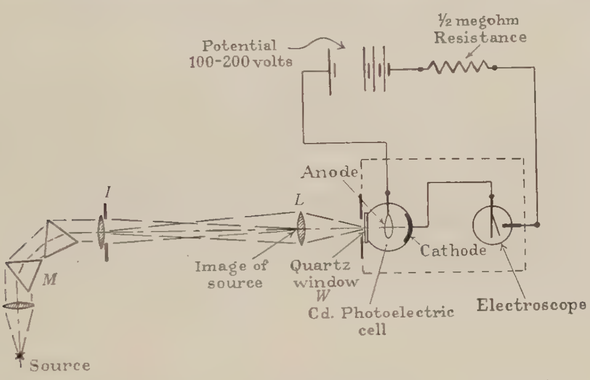

Fig. 1. General arrangement of apparatus employed.

When the cell is used for the comparison of relative energies for a given wave-length, the relation

$$P \propto \frac{1}{T} \times \frac{D^2}{d^2}$$

may be employed, where

P may be the power of the source,
D is the distance between the source (in this case I) and the cell,
d the diameter of cell aperture, and
T the time in seconds for one beat of the electroscope.

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* H. D. Griffith and J. S. Taylor, *Lancet*, **209**, 1205 (1925).

In practice five beats of the electroscope were taken, the timing being by stop-watch. *D* and *d* were constant throughout the measurements. The curve showing the sensitivity of the cadmium cell as a function of wave-length is given in figure 2.

A survey of the spark spectra of numerous metals reveals the fact that there are relatively few in which bright, single, and well-isolated lines are to be found. The most hopeful in this direction are those of Cd, Mg, Zn, Tl and possibly Al, spectrograms of which are shown in figure 3; and also that of the mercury arc in a silica tube. It will be seen that the zinc and cadmium spectra in the region between 0·25 μ and 0·29 μ have lines which are single and well separated, but none of them has the intensity of the line at 0·28 μ of the magnesium spectrum; unfortunately this line is a triplet and is not satisfactory in use except for adjustment purposes.

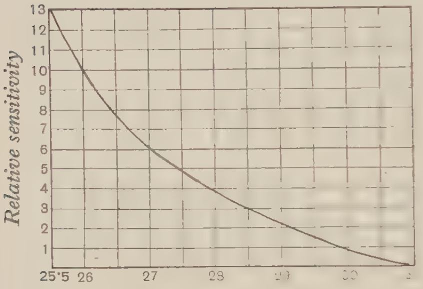


Fig. 2. Sensitivity curve for cadmium photo-electric cell.

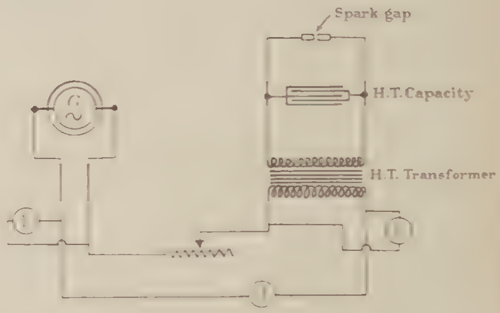


Fig. 4. Arrangement of electrical equipment for spark discharge.

The line of cadmium of wave-length 0·275 μ has been hitherto employed, and the ultra-violet objectives have been computed for it. There is, however, an excellent line at 0·257 μ, although it is not of such intensity as the former line; its isolated character should not be overlooked, for where a longer exposure can conveniently be given its strictly monochromatic nature renders it valuable in critical work. Other lines which might be useful at a future date for obtaining still higher resolution, although they are outside the range we are at present concerned with, are the cadmium line 0·227 μ and the zinc line 0·214 μ. The 0·185 μ line of aluminium is worthy of consideration as representing the shortest usable wave-length in air, and although it is of rather lower intensity than the above-mentioned lines, it offers possibilities for metallurgical work. Also there are lines in the spark spectrum of thallium which might be used, e.g. 0·277 μ, 0·253 μ and 0·230 μ.

Measurements of the relative brightness of the more suitable of these lines were taken, and the results below are given with the correction for the sensitivity of the cell to wave-length already applied. The spark was run under similar electrical conditions, shown diagrammatically in figure 4, throughout this test, namely 78 volts, 2·0 amp., 156 watts, and a capacity in parallel with spark of 0·005 μF. The length of the spark gap was 3 mm.

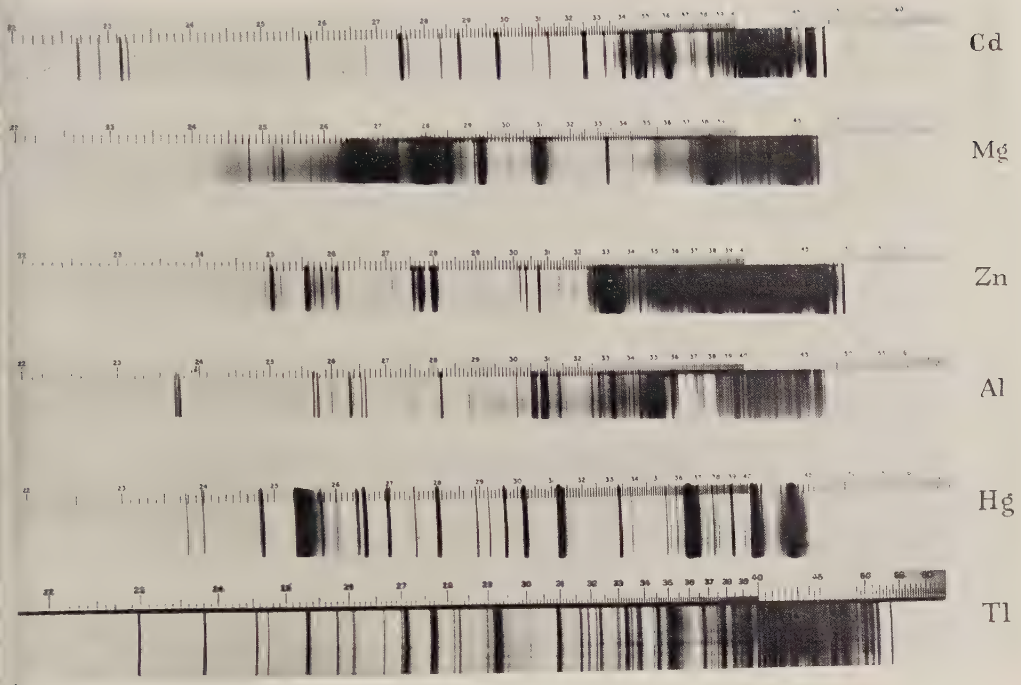


Fig. 3. Spectrograms of metals suitable for this work.

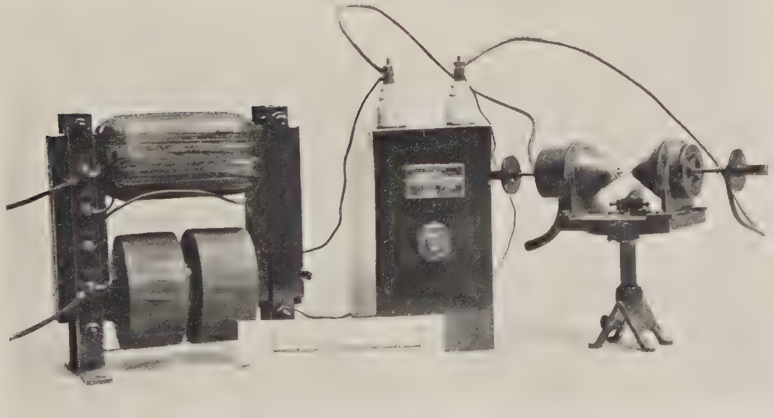


Fig. 10. Specially made transformer and condenser described in the text. Its compactness is shown by the scale.

Table 1. Intensities of spectral lines from 156-watt spark.

Metal	Wave-length (μ)	Relative energy $P = \frac{D^2}{Td^2}$ (arbitrary units)
Zinc	0·280 0·256	41·0 69·3
Cadmium ...	0·275 0·257	112·2 79·5
Magnesium ...	0·280 (triplet)	676·0
Aluminium ...	0·282 0·263	31·4 104·4
Thallium ...	0·277	63·0

From these values it is seen that the magnesium spark at $\lambda = 0{\cdot}28\,\mu$ is by far the brightest source to use in this region but, owing to the present impossibility of achromatizing the ultra-violet objectives even for a limited range of the spectrum, the group-like nature of this line, when used with the microscope, distinctly affects the definition of the image produced on the photographic plate. In figure 5, (*a*) and (*b*) show two photomicrographs taken with the 0·275 μ Cd line and the 0·28 μ Mg line respectively, and illustrate the point in question, showing the undesirability of using the latter line.

The next in order of intensity is the 0·275 μ Cd line, although it has only one-sixth of the luminosity; the aluminium line at 0·263 μ appears fairly strong, but the proximity of several other lines in this region detracts from its usefulness; the 0·282 μ line of the same metal is well isolated but low in intensity. The two zinc lines given in table 1 could be used but they are not particularly bright. The 0·277 μ thallium line would appear from the spectrograms to be distinctly useful, but as these latter are not of equal exposure the density of a line in one spectrogram cannot be compared with the same line in another; and unfortunately the intensity of this line turns out on photo-electric measurement to be relatively low.

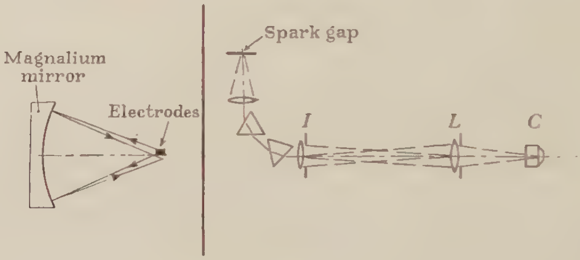
It was thought of interest to apply the test to a mercury arc as the source, for although some of its prominent spectrum lines were expected to be less intense than those of a spark emission, the convenience and steadiness in running of such a lamp commends its use very strongly. Table 2 below indicates the relative intensities, expressed in units similar to those used in table 1, of two lines in the named region, the voltage being 60 and the current strength 2·5 amp.

Table 2. Intensities of mercury lines from 150-watt arc.

Source	Wave-length (μ)	Relative energy
Mercury arc (quartz tube)	0·280 0·265	23·7 39·8

From this it is evident that in spite of the convenience of this type of source, the low intensity of the lines renders it of little use when short exposures are required; nevertheless the author on one occasion took an ultra-violet photomicrograph with the 2 mm. immersion monochromat using the Hg $0{\cdot}265\,\mu$ line with a perfectly satisfactory result, though the exposure needed was 4 minutes.

The question of steadiness of the source is of some importance, for it is easy to understand that if the image of the spectrum-line formed on the condenser of the microscope moves about due to fluctuations of the spark, it may happen that at any one instant the condenser and therefore the objective are not wholly filled with light, so that a consequent loss in resolution may occur. An experiment to test this possibility was made. The microscope was set up for visual light with a 2-mm. apochromat and a chemically deposited silver film as an object. Comparison photomicrographs were then taken, first with a pointolite lamp (tungsten arc) as source and then with a cadmium spark, the light in each case being rendered reasonably monochromatic ($\lambda = 0{\cdot}52\,\mu$) by the use of a filter, but all other conditions remaining the same. In figure 6 (*a*) and (*b*) show the results and indicate that the resolution is distinctly better when the steadier source is used, and therefore any attempt to increase the steadiness of the discharge at the spark gap would appear to be justified. Two plans have been employed in such an attempt. The first of these, due to a suggestion by Mr W. R. Bracey, was to use a concave magnalium mirror behind the spark gap at a distance such that the latter was situated at the centre of curvature of the mirror; by this means any upward or downward movement of the spark would be counteracted by a corresponding opposite movement of the spark image, figure 7 (*a*), and would thus tend to locate the source as a whole. The second method consisted in the use of an auxiliary quartz lens *L*, figure 7 (*b*), so placed in the plane of the spectrum produced by the monochromator as to form an image of the lens and iris *I* on the microscope condenser *C*. Thus the lens *I* acts in effect as the source of light. Both these devices have been found to operate excellently in practice and are to be recommended.



(*a*) (*b*)

Fig. 7.

Other points in the attempt to obtain better luminosity of the source, although of minor importance, are worthy of mention. One of these was the use of a reliable form of suction-pump for drawing off the fumes from the spark-housing. Readings taken with the photo-electric cell showed that the cloud forming between the spark gap and the collimator lens when the pump was not running accounted for a loss of about 35 per cent. of the incident radiation after a period of 4 minutes. Little or no advantage has been found in the oiling of plates at wave-length 0·275 μ. Various makes of photographic plates of different speeds have been tried, but it appears that a so-called "fast" plate for visible radiations differs very little in respect of exposure from a slow one at this wave-length. Hence it is advisable to use a small-grained plate, for instance a process plate, for this work.

Carbon disulphide has a fairly narrow transmission band in the region of $\lambda = 0{\cdot}275\,\mu$ (see figure 8). The possibility of using the Cd spark with such a filter directly instead of using the monochromator at once suggested itself, but unfortunately the transmission of the visible region of the spectrum precludes this—the hope of combining another filter to remove the visible and yet transmit the named ultra-violet region has not been realized*.

§4. ELECTRICAL CONDITIONS

The foregoing results indicate that the spark is the best source to use in order to increase the value B in the formula given on page 128, and also the 0·275 μ Cd line is the best to employ for this purpose. The next step, therefore, was to determine the most suitable arrangement of electrical conditions to give greatest luminosity of the spark with cadmium electrodes. Mr J. J. Holmes, a former research student in this department, has carried out valuable work in this direction with results shortly to be published. The author has had access to his paper which contains measurements and observations on: (*a*) single sparks produced by the charging of a large condenser to a high potential by an electrostatic machine; (*b*) series of sparks produced by means of an induction coil and mercury break; (*c*) series of sparks produced by means of an alternating current.

As it is this last section which has indicated the most hopeful line of approach to the desired aim, the writer has extended the work in this direction. A study of the spark intensity with variation of the three chief factors involved, namely (i) the frequency of the alternating current, (ii) the potential of the secondary current, and (iii) the capacity of the condensers, was made.

Apparatus. The electrical equipment consisted of (1) two alternators, one supplying nominally 500 watts at 100 ~ and the other 500 watts at 250 ~; (2) oil-immersed transformer (by Zenith Electrical Co.) at 100 volts and 0·25 amp, with step-up ratios of 100 : 1 to 200 : 1 variable by five intervals; (3) condensers of various types: (*a*) a variable pattern by Dubilier Co. ranging from 0·0027 to 0·008 μF, (*b*) a battery of tubular Leyden jars by Marconi Co., the individual capacities being 0·00067 μF, (*c*) a fixed type of 0·005 μF; (4) spark gap, with device for ensuring location and fixed width of electrodes; and (5) a special transformer, built to meet requirements suggested by the results of the experiments. The connexions were arranged as in figure 4, and as before the monochromator and photo-electric cell were employed as shown in figure 1.

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* With a film of carbon disulphide 0·25 mm. in thickness it was found that 54 per cent. of the incident radiation was transmitted at wave-length 0·275 μ, so that this liquid might possibly prove a valuable mounting medium for microscope objects.

Experimental work. The first test was made on the two alternators individually, to determine the effect of a change in frequency of the current. The second involved a change in the step-up ratio of the transformer, the capacity remaining the same throughout. Table 3 shows the figures obtained on these two tests.

Table 3. Effect of variation in frequency of current and change in step-up ratio of transformer.

Alternator	Primary			Step-up of transformer	Potential of secondary	Cycles per second	Capacity (μF)	Relative energy by photo-electric cell (arbitrary units)	Relative density by photographic method
	Volts	Amp	Watts						
100 ~	36	4	144	100 : 1	3600	86	·0048	70·8	0·93
	32	4	120	114 : 1	3648	83	·0048	61·4	0·91
	27	3·9	105	135 : 1	3658	79	·0048	54·5	0·72
	24	4	96	150 : 1	3600	74	·0048	46·6	0·66
250 ~	35	4·1	143	100 : 1	3500	228	·0048	68·6	0·92
	30	3·9	117	114 : 1	3420	217	·0048	60·9	0·90
	26	4·0	104	135 : 1	3523	204	·0048	54·1	0·72
				150 : 1	Too much load on alternator				

From these results it will be noticed that the change in frequency of the current, even when it comprises a threefold increase, has little effect on the apparent luminosity of the discharge. Also the potential of the secondary remains almost constant and does not increase with the increased step-up ratio of the transformer. (This is probably due to the overloading of the alternator and the consequent decrease in the power-factor.) There is, however, a direct correspondence between the intensity of the spark and the input wattage. As a check on these figures, photographs were taken with the ultra-violet microscope of a suitable object, and the relative density of the same part of the image was determined when successive exposures were made for variations in the electrical conditions as set out above. In the latter experiment all the exposures* were the same and all plates were given similar conditions of development. The densities were measured with a photo-electric density-meter†. Figure 9 illustrates this test, and the results obtained are appended in the last column of table 3; it will be seen that the relative-density values are in similar proportion to the figures given in the preceding column and thus bear out the results obtained by the first method.

Measurements were then taken with the same step-up ratio of the transformer but with a variation in the amount of capacity put in parallel with the spark. Table 4 shows the values obtained.

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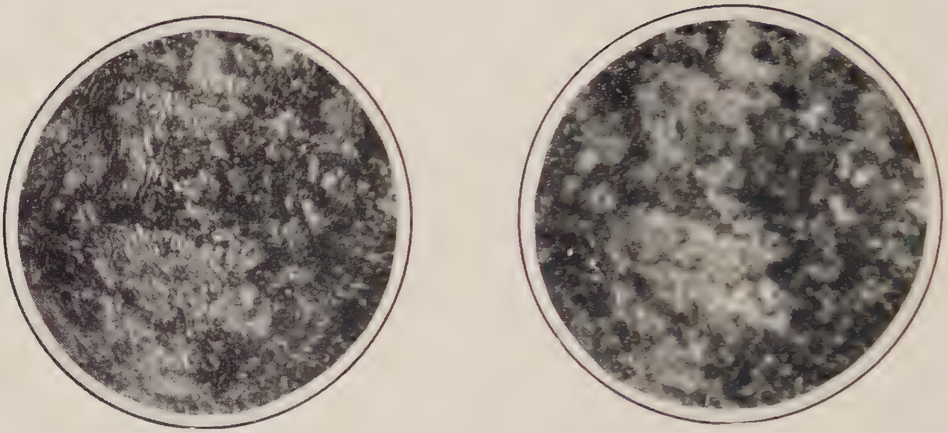
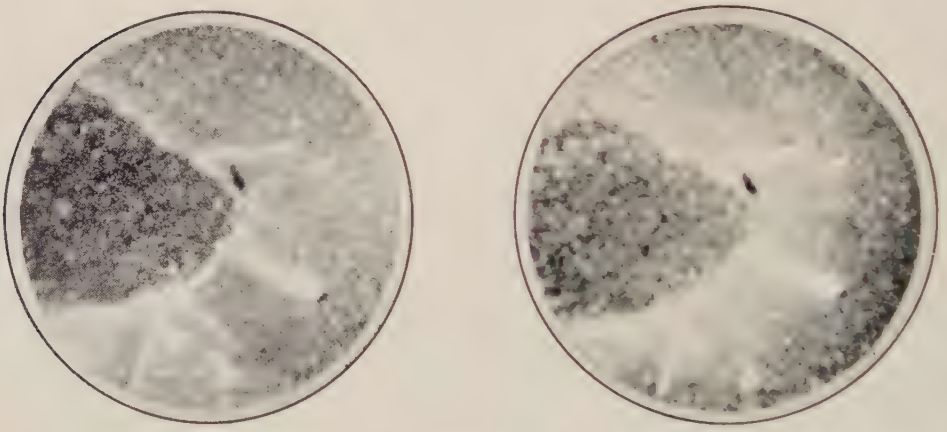
* 10 sec. by stop-watch. This exposure was found to give densities within the range of Schwarzschild's law.

† F. C. Toy, *Journ. Sci. Inst.* **4**, 369 (1927).



(*a*) (*b*)

Fig. 5. Contrast in definition when using (*a*) Cd 0·275 μ line, and (*b*) Mg 0·28 μ line.



(*a*) (*b*)

Fig. 6. The effect on resolving power of using (*a*) steady, (*b*) unsteady sources of illumination.

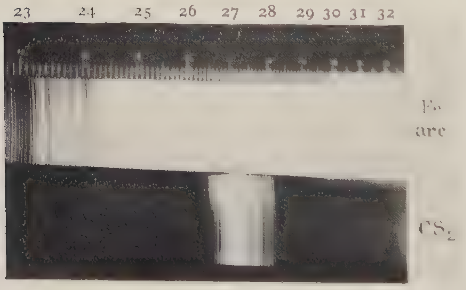


Fig. 8. Transmission of CS_2 in the ultra-violet.

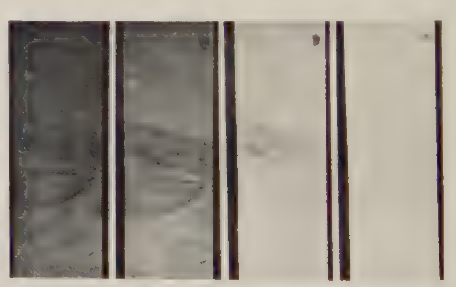


Fig. 9. Change in photographic density with variation of electrical conditions.

From these figures it would appear of considerable importance to increase the amount of capacity to as large a value as possible in order to increase the brightness of the spark. It will again be noted that the frequency of the current has little bearing on the brightness.

Table 4. Effect of change in capacity of condensers.

Alternator	Primary			Step-up of transformer	Potential of secondary	Cycles per second (actual)	Capacity (μF)	Relative energy by photo-electric cell	Relative density by photographic method
	Volts	Amp	Watts						
100 ~	30	3·9	117	100 : 1	3000	85	·0027	47·2	0·68
	30	4·0	120	100 : 1	3000	82	·0037	53·1	0·73
	30	4·0	120	100 : 1	3000	83	·0048	63·0	0·92
	35	4·1	143	100 : 1	3500	81	·0059	79·6	1·02
250 ~	38	4·0	152	100 : 1	3800	230	·0027	48·2	0·70
	38	4·0	152	100 : 1	3800	228	·0037	54·8	0·74
	38	4·0	152	100 : 1	3800	232	·0048	62·5	0·91
	39	4·1	157	100 : 1	3900	229	·0059	78·1	1·01

From the foregoing tests it was apparent that the two most important factors in the improvement of intensity were, firstly, increased input energy, and secondly, high capacity. Steps were therefore taken to increase the wattage hitherto used, namely 150; and a special transformer*, which enabled a larger input energy to be used by reason of an improved power factor, was employed. The particulars of this transformer are as follows: Primary volts 100, amp 5, cycles per second 100. Step-up ratio 50 : 1. Secondary volts 5000, amp. 0·1, kilowatts 0·5.

With this equipment the series of experiments was repeated, but in this case the capacity was gradually increased until the spark refused to jump the gap of 3 mm. Table 5 shows the results obtained.

If we compare line 3 of these results with those in table 3, where the capacity is similar, we see, firstly, that by increase of the input energy the intensity of the spark has been considerably increased. Actually, however, although the wattage is now five times what it was in line 3 of table 3, the relative energy appears only to be trebled; nevertheless this tends to agree with indications given by a graph of the results of table 3, and in any case a distinct increase in brightness has resulted. Secondly, the marked progressive increase in energy with increase of capacity is evident. It will be noted that with the electrical conditions as given in the last line but one in table 5 the brightness of the Cd spark has been increased to nearly six times that obtained at the commencement of these experiments. In practice, however, the discharge is so violent that the cadmium electrodes burn away very rapidly; it is nevertheless possible to use the spark gap under these conditions for about 5 seconds, which, with the increased luminosity thus obtained, is considerably longer than the time now required for an exposure with an object of the transparent type. During all adjustments prior to the actual exposure the wattage in the primary is limited by suitably controlled resistances, long usage of the electrodes being thus made possible.

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* Designed jointly by Messrs H. Gough, G.I.E.E., and G. K. Johnson, A.M.I.E.E., A.M.I.M.E., and made in experimental form by Messrs Watson and Sons (Electro-medical), Ltd.

Table 5. New transformer tests.

Primary			Step-up of transformer	Cycles per second	Capacity (μF)	Relative energy by photo-electric cell
Volts	Amp	Watts				
100	5·0	500	50 : 1	100	·0027	84·5
,,	,,	,,	,,	,,	·0037	117·6
,,	,,	,,	,,	,,	·0048	169·0
,,	,,	,,	,,	,,	·0059	208·0
,,	,,	,,	,,	,,	·0069	245·8
,,	,,	,,	,,	,,	·0077	300·4
,,	,,	,,	,,	,,	·0088	314·4
,,	,,	,,	,,	,,	·0098	338·0
,,	,,	,,	,,	,,	·0109	365·4
,,	,,	,,	,,	,,	·0120	409·0
,,	,,	,,	,,	,,	·0131	Would not spark across gap

An ultra-violet photomicrograph taken with the 2-mm. immersion monochromat at a magnification of 840 × with an exposure of only 1 second is now found to give a negative of 0·69 density, a reasonably dense photograph, whereas previously a similar exposure produced a density of 0·11—a value which is too low to reveal detail in the photomicrograph.

§ 5. CONCLUSIONS

The impressions received from this work are: (i) that the spark discharge between cadmium electrodes is the most suitable source of radiation for the quartz monochromat microscope objectives computed for and used with a wave-length in the neighbourhood of 0·275 μ; (ii) that it is still necessary, in spite of considerable loss of light, to employ a monochromator for providing the desired wave-length; (iii) that the intrinsic brightness of the spark appears to be a function solely of the energy-input and the capacity employed. Furthermore the frequency of the alternations of the current seem to be of little importance. In this latter connexion the transformer can be used with alternating mains at 50 ~ so as to save the expense of an alternator; (iv) that a compact and inexpensive electrical unit (see figure 10) can be designed for providing the source of illumination for ultra-violet microscopy.

§ 6. ACKNOWLEDGMENTS

I wish to acknowledge the continued and valuable help of Mr G. K. Johnson, and to express my thanks to Mr H. Gough for his collaboration in the design and manufacture of the new transformer and condenser, and to Professor L. C. Martin for his encouragement and advice throughout the work.

DISCUSSION

Mr T. Smith suggested that, in view of the difficulties incident to illumination with an extended source, the author should give an idea of the numerical aperture used, for instance, in the production of figure 6. It was rather a pity to include the debatable formulae given on pages 2 and 3: it would be sufficient for the author's purpose to assume merely that the illumination varies as the intrinsic brightness and as the frequency of the beats of the electroscope. The table on page 128, showing the distribution of energy in the spectrum, was not quite to the point: the information required was the energy in particular lines, not regions of the spectrum. The preceding statement that the maximum lies in the visible green of the solar spectrum needed qualification; it was only true on a wave-length basis, whereas if the spectral energy were plotted on a frequency basis the maximum would lie in the infra-red.

Mr J. H. Awbery. In regard to Mr Smith's remarks, it seems to me that we are not comparing the energy in an arbitrary strip of the spectrum. If the energy in a given spectral line, or in the range called "green," is under discussion, then when the basis of calculation is changed from wave-length to frequency, the range considered has to be altered in such a way that the area (i.e. the energy) is the same whichever basis is adopted.

Prof. L. C. Martin. I think the author is to be congratulated on a piece of work which has led to such practically useful results. The result of essential value is the dependence of the effective intrinsic brightness of the spark on the energy or wattage consumed, and its practical independence of the frequency. I had hoped that some additional research work done in the Technical Optics Department by Mr J. H. Holmes would be described at this meeting, but the paper could not be prepared in time. It will throw considerable light on the above result. With regard to the formula used in connexion with the photo-electric cell, I regarded this at first with some suspicion, but a series of practical tests showed that it is quite a close approximation.

Author's reply. The photomicrographs in figure 6 were taken with a 2 mm. apochromal objective of numerical aperture 1·2. The formula on page 128, besides showing the way in which the illumination per unit area may vary, also brings out the importance of keeping the magnification as low as its minimum limit will permit, and therefore the inclusion of this relation would appear to be justified. The table referred to is intended to convey an idea as to the distribution of energy in the spectrum given by a spark emission. The enhanced nature of lines in the ultraviolet region of the spark spectrum is likely to make this form of source more useful than one, such as for example the arc or flame, in which the lines of longer wavelength may be, generally speaking, of greater relative intensity.

THE INFLUENCE OF THE CRYSTAL-ORIENTATION OF THE CATHODE ON THAT OF AN ELECTRO-DEPOSITED LAYER

By W. A. WOOD, M.Sc., The National Physical Laboratory

Communicated by Dr G. W. C. Kaye, October 29, 1930.
Read and discussed December 5, 1930.

ABSTRACT. The influence of the crystal-orientation of a cathode on that of an electro-deposited layer is studied by X-ray methods for the cases of copper and nickel, respectively, deposited on rolled copper. The conditions of cathode-surface and current-density which accompany an oriented deposit are determined. The orientation of the copper deposit for small currents is the same as that of the cathode. The nickel, at low current densities, assumes a distinct orientation. As the current is increased there is a region of no orientation, followed, at still higher currents, by an orientation the same as that of the cathode surface below.

§ 1. INTRODUCTION

The very interesting work of a group of investigators* has shown that the minute crystalline grains of an electro-deposited layer tend under certain conditions to arrange themselves in such a way that one particular crystallographic axis sets itself normal to the cathode face. The deposit is said to have assumed an orientation of the fibre type. It becomes, therefore, a matter of further interest to extend the study to the case of deposition upon a surface which, as a result of previous working, itself possesses some degree of orientation. Many materials are in this state in practice, and the relative orientation of layer and base might very reasonably be expected to be a factor of some influence on the properties of the final product. The methods of X-ray analysis provide the most direct approach. They are used in the present work, which deals with the problem of copper and nickel deposited on oriented copper sheet.

§ 2. COPPER ON COPPER

(i) *Experimental*. The material used as cathode was obtained from sheet copper which, after having been rolled to various extents, gave samples possessing different degrees of orientation. This orientation was shown by means of X-ray photographs taken with iron K_α radiation to begin immediately at the surface. This precaution is a necessary one, as the author has shown† that in the case of a wire the action of the die in drawing tends to destroy surface orientation. An anode copper sheet was used. Traces of orientation were removed from the anodes by heat treatment.

The plating bath contained per litre 200 gm. of copper sulphate and 100 gm. of sulphuric acid. In certain cases, as described below, the cathodes were chemically cleaned. This process involved first washing the surface with benzene and alcohol and then placing the specimen as cathode in a solution containing 60 gm. of sodium carbonate, 7 gm. of caustic soda and 7 gm. of sodium cyanide per litre. After being subjected to cathode bombardment for 1 minute the specimen was withdrawn and rinsed in distilled water. Next it was "pickled" by immersion for some 20 seconds in dilute nitric acid. Finally after another rinsing it was hung directly in the plating bath, thereby completing the circuit*. No stage of the process appreciably affected the orientation of the copper.

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* G. L. Clark, *Applied X-rays*, p. 241.

† Paper not yet published.

In each experiment two exactly similar baths were connected in series. In the first the cathode had been cleaned, and in the second not cleaned. The current-density was calculated from the area immersed and the total current passing.

The difficulty of detecting which lines in the X-ray spectrum were due to the original copper and which were due to the deposit disappeared with the observation that, in the region of current-densities employed, the deposited grains produced the characteristic spotted effect which always arises from the presence of crystallites larger than approximately 10^{-3} cm. The interference lines consisted therefore of spotted lines caused by the deposit superimposed upon the smooth continuous lines caused by the original copper. The spectrum of the latter was, of course, weakened by absorption in the layer to an extent depending on the thickness of that layer. In most of the actual experiments the deposit was so thick that the lines due to the original copper were entirely absorbed. It was found very convenient, moreover, to place the specimen at such an angle to the incident X-ray beam that the lines in the spectrum were somewhat out of focus. The sharp diffraction spots due to the layer were thus spread over the increased width of the corresponding lines, and, being therefore distributed over a greater area, they were more easily distinguishable. A "bad" photograph was thus preferable to a good one in which the focussing would cause a concentration of the spots into a line almost as uniform as that from the original copper. Every photograph was taken under the same geometric condition.

(ii) *Observations.* It was found that only in the case of the chemically cleaned surfaces did the electrodeposited layer assume an orientation similar to that of the base. This fact could be utilised as another method of distinguishing the spectrum of the deposit from that of the base metal. Each cleaned specimen was accompanied by one not specially cleaned and both specimens had taken the deposit under exactly the same conditions of time, current-density and solution. Consequently, when the spectrum of one deposit showed orientation and the other none at all, neither spectrum could be put down to the original oriented copper. Apparently the layer of dust and grease present on normal material introduces complications sufficient to prevent the deposit following the orientation below.

The effect depended on the current-density. Using the electroplating bath described above one found that at current-densities greater than 12 mA/cm.^{2} the effect began to disappear and at 15 mA/cm.^{2} the deposited grains were arranged entirely at random. Increasing the current had no effect other than reducing the size of the grains. At 60 mA/cm.^{2} the spotted effect of the grains had gone. Below 12 mA/cm.^{2}, however, the grains were deposited in a way which showed strong orientation. This orientation was the same as that of the copper surface below. These results are illustrated by figures 1, 2, 3. In figure 1 is reproduced the photograph of the original oriented cathode; the lines are not focussed. Figure 2 shows the spectrum of a deposited layer. It can be seen that the diffraction spots due to the deposit concentrate on the intensity maxima shown in the previous figure and absent themselves from the minima. The deposit is oriented like the original. Figure 3 is the photograph of a deposit on an unclean surface and it may be seen that here the spots are scattered equally along the lines. The orientation has gone. For those values of the current-density which permitted the effect to occur, it was found that the time of deposition could be extended to at least 24 hours without causing any appreciable decrease in the degree of orientation. The orientation appeared likely to persist for the thickest layers obtainable under the conditions described.

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* W. E. Hughes, *Modern Electroplating*, chap. 3 and 10.

The introduction of a colloid in the form of gelatin decreased the grain size without, as it seemed, affecting the orientation. The only indication that the observed spectra, now unspotted, were not due to the original specimen, however, was that the comparison photograph from the unclean cathode showed no orientation.

§ 3. NICKEL ON COPPER

(i) *Experimental.* The plating solution contained per litre 300 gm. of nickel sulphate, 6 gm. of boracic acid and 3 gm. of common salt*. As in the experiments on copper, deposition was made under similar conditions on cleaned and unclean copper strip. A piece of sheet nickel used as anode showed no trace of orientation. The identification of the interference lines was simpler than in the case of copper, the pinakoidal spacings of copper being 3·603 A.U., and of nickel 3·499 A.U. Copper K_α radiation was used.

(ii) *Results.* The interference lines of the nickel plate were unbroken; this shows that the deposit possesses the fine-grained structure characteristic of that metal. The effect of time of deposition and of the cleanliness of the surface were the same as for copper.

The orientation of the deposit depended on the current-density in a curious manner. It was observed that for current-densities less than 7·5 mA/cm.^{2} the deposited nickel was very highly oriented. The orientation, however, was different from that of the copper surface. It was of the fibre type and would agree with that found by Clark and Fröhlich† in their work on non-oriented cathodes. The strong maximum in figure 4 illustrates this type of orientation. In that particular photograph the deposit was obtained with a current-density of 6 mA/cm.^{2} passed for 8 hours. At a current-density of 7·5 mA/cm.^{2} the deposit showed no orientation whatsoever. Figure 5 reproduces a photograph of the deposit at this stage. The absence of orientation persisted, moreover, for current-densities up to 10 mA/cm.^{2}.

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* W. E. Hughes, *loc. cit.*

† G. L. Clark and P. Fröhlich, *Zeit. Electrochemie*, **31**, 655 (1925).

Fig. 1.

Fig. 2.

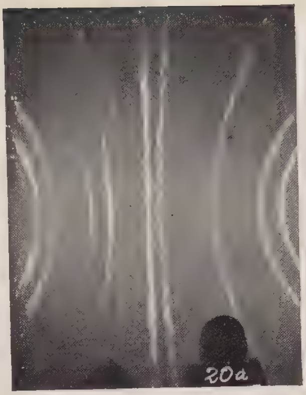

Fig. 3.

Fig. 4.

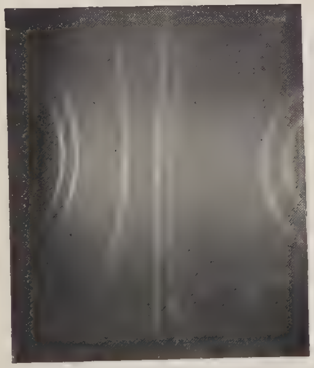

Fig. 5.

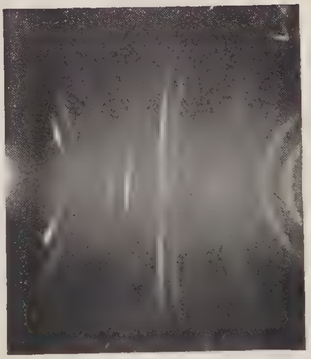

Fig. 6.

For current-densities up to about 40 mA/cm.^{2} the deposit again showed a high degree of orientation. In this case the orientation was the same as that of the original copper surface underneath and, but for the slightly different spacings of nickel, would hardly have been distinguishable from it. Figure 6 shows a typical photograph of a deposit made in this region of current-density. Finally, for still higher current-densities the orientation again tended to disappear and was entirely absent at 60 mA/cm.^{2}. In all the nickel photographs shown the differences of current-densities were so combined with the time of deposition as to give as nearly as possible the same thickness of deposit. Similar results were found for deposition on wires.

§ 4. DISCUSSION OF RESULTS

From the above observations it would appear to be a simple matter to produce a marked orientation in an electrolytic deposit. The current-density must not exceed a certain value such that the ions, as they deposit themselves, are given enough time to take up the orientation. The surface must be clean so as to exclude the possibility of an interposed layer of foreign matter. The orientation actually assumed by an ion depends on the lines of force due to the potential difference between the electrodes and on the localized fields of force due to the particular arrangement of the original atoms on the surface. In the experiments with a copper deposit the latter forces always predominate and the deposit repeats the underlying orientation. In the case of nickel the former forces predominate at very low current-densities and the latter at high current-densities, while for intermediate current-densities the two fields apparently compensate one another and produce a random result. The local forces due to the original orientation are presumably the same for any current-density, so that one must conclude that at the higher currents the influence of the force due to the potential across the electrolyte on the depositing ions is decreased.

The author hopes to extend the work to other metals and to correlate the results with the physical properties of the various deposits.

§ 5. ACKNOWLEDGMENTS

In conclusion, the author wishes to express his thanks to Mr J. R. Clarkson, B.Sc., for his efficient help in the experimental work, and to Dr G. Shearer for his interest in the research.

DISCUSSION

Mr J. Guild asked whether the optical properties of the metallic films are affected by the orientation of the crystals. For instance, did the reflecting-property of the silver or platinum films vary with this factor?

The Author said that he had not had facilities for testing this interesting point. Prof. Carpenter had found, however, that the reflecting-properties of single crystals are different for different faces.

THE INFLUENCE OF LOW TEMPERATURES ON THE THERMAL DIFFUSION EFFECT

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ABSTRACT. Measurements of thermal diffusion on the following gas mixtures at temperatures between 15° C. and – 190° C. are described: He—Ne; H_2—Ne; He—A; Ne—A; He—N_2. The experiments confirm and extend the authors' previous work showing that there is a general tendency for k_t to decrease at low temperatures. It is found, however, that for the pairs of gases He—Ne and H_2—Ne, with low liquefying points, the change in k_t between 15° C. and – 190° C. is small. Chapman's theory is used to deduce approximate values of the laws of repulsive force operating between unlike molecules during collisions. At low temperatures molecules tend to become "softer" and their behaviour is less like that of rigid elastic spheres. The "hardest" molecules appear to show the smallest variation in k_t. The influence of helium in this respect, on the value of k_t for a mixture, is clearly shown.

§ 1. INTRODUCTION

An account of a series of experiments on thermal diffusion* at low temperatures was given in a recent paper†. Mixtures of gases were examined with one side of the apparatus at about 15° C. and with the other at temperatures down to – 190° C. It was found that the thermal separation is not always proportional to log (T_1/T_2), T_1 being the absolute temperature of the hot side and T_2 that of the cold side. For the lower values of T_2 the thermal separation falls below the proportional value, which means that k_t, the ratio of the coefficient of thermal diffusion to the coefficient of ordinary diffusion, decreases with temperature. Lugg‡ has recently shown that k_t for a mixture of hydrogen and carbon dioxide increases at high temperatures. These changes in the value of k_t indicate the possibility of changes in the nature of inter-molecular collisions—i.e. the molecules tend to become "softer" at low temperatures; and it was suggested that this might be due to the influence of the attractive forces between molecules. In particular, it was shown that the falling off in k_t for nitrogen-argon mixtures was greater than for such mixtures as hydrogen-nitrogen, or hydrogen-argon. It is therefore of

T_1, T_2

k_t

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* D. Enskog, *Phys. Zeit.* **12**, 538 (1911); *Ann. d. Phys.* **38**, 742 (1912). S. Chapman, *Proc. R. S.* A, **93**, 1 (1916); *Phil. Trans.* A, **217**, 157 (1917); *Phil. Mag.* **34**, 146 (1917); **38**, 182 (1919); **48**, 602 (1924); *Proc. R. S.* A, **119**, 34 (1928); A, **119**, 53 (1928); *Phil. Mag.* **7**, 1 (1929). S. Chapman and F. W. Dootson, *Phil. Mag.* **33**, 268 (1917). T. L. Ibbs with a note by S. Chapman, *Proc. R. S.* A, **99**, 385 (1921); A, **107**, 470 (1925). T. L. Ibbs and L. Underwood, *Proc. Phys. Soc.* **39**, 227 (1927). T. L. Ibbs, K. E. Grew and A. A. Hirst, *Proc. Phys. Soc.* **41**, 456 (1929). G. A. Elliott and I. Masson, *Proc. R. S.* A, **108**, 378 (1925). J. W. H. Lugg, *Phil. Mag.* **8**, 1019 (1929).

† *Proc. Phys. Soc.* (1929), *loc. cit.*

‡ *Phil. Mag.* (1929), *loc. cit.*

interest to examine at low temperatures mixtures of a pair of gases for which the attractive forces are regarded as weak. Mixtures of helium-neon and of hydrogen-neon satisfy this condition; the general physical behaviour of these gases indicates a weak attractive field. This conclusion is supported by the theoretical investigation of Lennard Jones*, who has shown that the molecules of these gases can be nearly represented by point centres of repulsive force. For these pairs of gases and for other pairs now examined it has been necessary to work with small quantities of the gases. This has necessitated considerable modification of our experimental methods.

Prof. S. Chapman had previously pointed out to us the need for measurements of k_t for pairs of monatomic gases, and had very kindly obtained the valuable co-operation of Dr F. W. Aston, who devoted considerable time to demonstrating to one of us (T. L. I.) the methods of manipulating and purifying small quantities of gases. Dr Aston allowed us to have full details of the apparatus he has employed for the purpose, and also lent quantities of the rarer inert gases. We have thus been enabled to make measurements on a number of pairs of the monatomic gases which are of special interest owing to the relative simplicity of their molecular structure.

A preliminary investigation was made which showed that our methods could be adapted for use with small quantities of gases. In this early work experiments were made on a number of pairs of inert gases by a comparative method, with the hot side at 100° C. and the cold side at room temperature. In the meantime our work at low temperatures had reached the stage described, where it appeared desirable to develop the method so that measurements could be made on helium and neon. We decided therefore to combine the two series of experiments so as to obtain not only the ordinary value of k_t but also its variation with temperature.

§ 2. METHOD OF EXPERIMENT

Some of the more important parts of the apparatus are shown in figure 1. The "flow" method previously employed, which required considerable quantities of gas, was discarded. The gas mixtures were made in the gas burette† *B*, of capacity about 80 cm.^{3}, which communicates by a glass tube with the diffusion apparatus shown on the right-hand side of the diagram. The glass bulb *A* of volume 25·7 cm.^{3} is joined by a connecting tube *C* of length about 1·0 cm. and internal diameter 0·3 cm. with the katharometer‡ *K*. Taps T_3 and T_4 communicate with glass bulbs, one of which contained charcoal, used in the purification of the gases. Mr G. O. Harrison gave valuable assistance by his skill in the construction of the apparatus.

The katharometer is employed as before to analyse the gas mixtures and to measure the changes in composition produced by thermal diffusion. The bridge arrangements were the same as those previously described§ and are not shown in the diagram. To obtain a calibration curve for any pair of gases a quantity of one of them is drawn into the burette through the syphon tube S and the mercury level in the burette is adjusted to coincide with the point of the glass index I. The pressure of the gas in the burette can be obtained by observation of the difference of levels of the mercury at I and in the reservoir R, and the height of the barometer. A quantity of the other gas is then drawn in and the pressure of the mixture obtained when the volume is the same as before. In this way a mixture of known composition by volume is made. The mixture is then passed through taps T_2 and T_5 into the katharometer and bulb A. The level of R is adjusted to bring the gas to atmospheric pressure and the galvanometer reading for the mixture is observed. By drawing further quantities of gas into the burette the calibration curve for mixtures of a pair of gases is thus obtained, and by reference to this curve any other mixtures of the pair can be analysed.

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* J. E. Lennard Jones, *Proc. R. S.* A, **107**, 157 (1925).

† Cf. Travers' *Study of Gases*, p. 67.

‡ This instrument was kindly lent by Dr G. A. Shakespear.

§ *Proc. R. S.* A, **107**, 475 (1925).

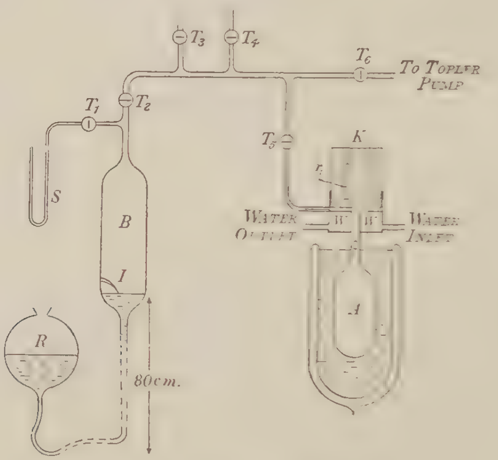

Fig. 1. Diagram of apparatus.

The thermal diffusion measurements. A mixture is passed into the diffusion apparatus at atmospheric pressure as described and tap T_5 is closed, the bulb A being at room temperature. This bulb is now surrounded by a Dewar vessel containing liquid air or cooled pentane, and thermal diffusion is allowed to take place. This causes in the concentration of the gas on the hot side a change which is measured by means of the katharometer. The total volume of the hot side, i.e. the katharometer cell and its connections, is 0·98 cm.^{3} and this side is kept at a steady temperature by the water jacket W supplied from the mains. The small change in composition which occurs on the cold side can be calculated and the total separation obtained. When the gas in A is cooled the pressure is reduced, but previous experiments* have shown that thermal diffusion is not affected by such changes of pressure. By using this fact we have been able to reduce considerably the manipulation required in the experiments, as the same gas mixture can be allowed to remain inside the diffusion apparatus for a series of measurements with the cold side at

* Cf. *Proc. Phys. Soc.* **41**, 468 (1929).

different temperatures. (In practice the bulb A was allowed to warm up to room temperature by removal of the Dewar vessel before each measurement, so that the katharometer zero could be checked. This procedure avoids small errors which may occur owing to fluctuations of the current in the bridge circuit or to changes of temperature in the water jacket.) The changes in pressure produce small changes in the katharometer readings, but the necessary correction for these can be made for each mixture. The correction is obtained by observation of the effect of changes of pressure on the galvanometer zero when the diffusion apparatus is at room temperature. It is worth noting also that if all the diffusion measurements had been made at atmospheric pressure considerably greater quantities of gases would have been required for the experiments at low temperatures. At the end of a series of measurements on one mixture the gas is removed through tap T_6 by means of a Töpler pump and collected over mercury. The gas can then be used again in another mixture. Each pair of gases was generally examined in mixtures of three different proportions.

Temperature control and measurement. Measurements at the lowest temperatures were made as before by use of liquid air in the Dewar vessel. Temperatures down to about − 145° C. were obtained by means of pentane cooled with liquid air. To obtain uniformity of temperature the pentane was again stirred by bubbling compressed air through it. The shorter glass bulb A now used was unsuitable for our previous method of obtaining temperatures between − 145° C. and − 190° C. The absence of observations over this range does not in this case produce any serious difficulty as our previous investigation has shown the general nature of the effect.

Greater precision in the temperature-measurements was obtained by use of a thermocouple instead of a pentane thermometer to measure the cold side temperature. The couple was of copper and copper-nickel alloy ("Ferry" wire). The junctions were silver-soldered. One junction was immersed in ice and water in a Dewar vessel and the other suspended in the pentane near the middle of the diffusion bulb A. The thermo-electric e.m.f. was balanced by the potentiometer method, and the potentiometer readings were calibrated by direct comparison of the thermocouple with a constant-volume hydrogen thermometer. The temperature of the hot side was again measured with a mercury thermometer placed in the outlet of the water bath.

Precision of the measurements. The error in the absolute values of the separation now recorded is probably of the order of about 2 or 3 per cent., while relatively to one another the values for any one mixture are correct to about 1 per cent. The fact that there was now no flow of gas through the apparatus improved the steadiness of the katharometer readings.

An alteration in the mean composition of the gas in the connecting tube to which the temperature gradient is applied occurs as the thermal diffusion effect progresses. With the present arrangement the proportion in the tube of the lighter gas increases with the effect, which will cause a change in the value of k_t. The correction required to make k_t apply throughout to the original mixture has not yet been made in these results, but it may have to be considered as the theory is further developed. Many of the mixtures examined contained less than 50 per cent. of the lighter gas, and the change of composition in the connecting tube would therefore tend to make the observed separation at low temperatures greater than the true separation for the original mixture, i.e. the falling off in k_t is actually more than is shown by the curves.

Purification of the gases. The gases used were purified in the apparatus by fractionation with charcoal and liquid air. The purity of a gas for our purpose can be examined by testing of the gas, without admixture, for any evidence of the thermal diffusion effect. This provides a very convenient method of testing the purity of the inert gases, as a small quantity of an impurity of considerably different molecular weight—e.g. small quantities of helium as an impurity in neon—can easily be detected.

In our previous work at low temperatures we came to the conclusion that the simple gas conditions postulated in Chapman's theory could be regarded as holding in the conditions of the experiments. Certain experimental tests of this conclusion were then made, and we consider that it is also valid for this work.

§3. THE RESULTS OF THE EXPERIMENTS

Five pairs of gases were examined: (1) helium and neon, (2) hydrogen and neon, (3) helium and argon, (4) neon and argon, (5) helium and nitrogen. It is convenient to demonstrate the results in the form of graphs showing the relation between separation and $\log_{10}(T_1/T_2)$, as the characteristic behaviour of different gases can be seen from these curves. All the mixtures were examined down to − 190° C.; the points on the first part of the curves are obtained by means of cold pentane, and the point at the end by means of liquid air. As the temperature T_1 for any mixture is practically constant, the value of k_t in a region at a temperature T_2 is proportional to the slope at the corresponding point on these curves and can be calculated from it*.

By means of the formulae of Chapman and Hainsworth† an expression can be obtained, for a pair of gases, for k_t as a function of the proportions of the gases in the mixture, the molecules being regarded as rigid elastic spheres. As predicted by Chapman, the experimental values of k_t are smaller than the calculated values for rigid elastic spheres, and we can obtain a "thermal separation ratio" R_t, where

R_t

$$R_t = (k_t \text{ experimental})/(k_t \text{ for rigid elastic spheres}).$$

The value of R_t appears to be nearly independent of the proportions of the gases in the mixture and can therefore be regarded as a constant for a pair of gases. (A more careful investigation of the constancy of R_t will be made in due course.) Thus when R_t is known it can be used in conjunction with Chapman's formulae to calculate the thermal separation in any practical case. Outside a certain range of temperature R_t will of course vary with temperature, but for all pairs of gases there will be an ordinary value of R_t which can be applied over a considerable temperature range. A knowledge of this may be of value, as thermal separation must frequently occur in experimental and practical circumstances where gas mixtures are used.

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* Cf. *Proc. Phys. Soc.* **41**, 463 (1929).

† *Phil. Mag.* **48**, 602 (1924).

The different mixtures of any pair of gases which have been examined are lettered *A*, *B* and *C* for reference purposes both in the text and in the diagrams. The proportions of the gases in each mixture are given in the tables at the end of the paper.

(1) *Helium and neon.* The results for the three mixtures examined are shown in figure 2. The points obtained by means of pentane, i.e. down to – 145° C., lie on a straight line, and the liquid-air point in two cases lies practically on this straight line extended, and in the third case a little below it. This means that the experimental value of k_t for a helium-neon mixture can be regarded as nearly constant from 15° C. to – 190° C. This result is of interest as it is the only one of the kind obtained throughout this range of temperature. The correction for the change in composition of the gas in the connecting tube would produce a slight curvature and tend to reduce k_t a little at the lower temperatures. Both helium and neon have low liquefying-points, and it may be worth while later to study them more fully, particularly in the range from – 150° C. to – 190° C.

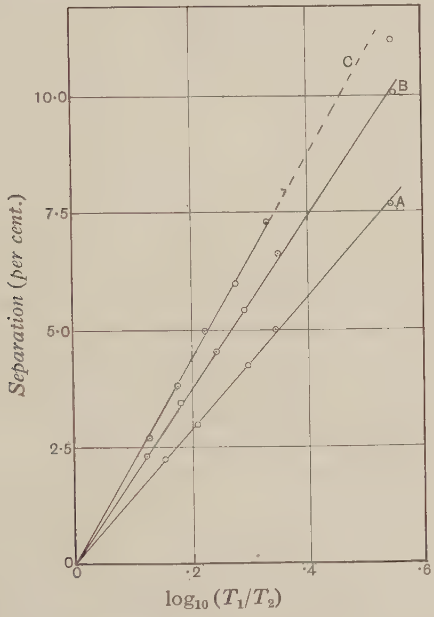

Fig. 2. Helium-neon. Mixtures *A*, *B* and *C*.

Using Chapman's formulae we obtain for this pair

$$k_t = \frac{5}{2}\left\{\frac{0{\cdot}255\lambda_1 + 0{\cdot}313\lambda_2}{3{\cdot}062 + 1{\cdot}713\lambda_1/\lambda_2 + 1{\cdot}046\lambda_2/\lambda_1}\right\},$$

where λ_1 and λ_2 are the proportions by volume of the heavier and lighter gases respectively in the mixture, so that $\lambda_1 + \lambda_2 = 1$.

λ_1 λ_2

This gives to k_t, for the three mixtures *A*, *B* and *C* examined, the respective values 0·080, 0·110 and 0·124. The experimental values of k_t obtained from the curves are 0·061, 0·080, 0·093, so that R_t has the values 0·77, 0·73, 0·75. This gives a mean value of R_t for the pair of about 0·75, which can be regarded at present as nearly independent of temperature over the range which we have studied.

(2) *Hydrogen and neon.* In the three mixtures examined the proportionality of separation and $\log (T_1/T_2)$ holds over the range of temperature from 15° C. to − 145° C. The separation obtained by means of liquid air is in all cases definitely below the proportional value: this is shown for two of the mixtures in figure 3, the liquid-air point lying below the straight line produced. The falling off is however small, as the separation with liquid air is only about 8 per cent. below the proportional value. This compares with a falling off in separation of about 20 per cent. previously recorded* for mixtures of hydrogen and nitrogen.

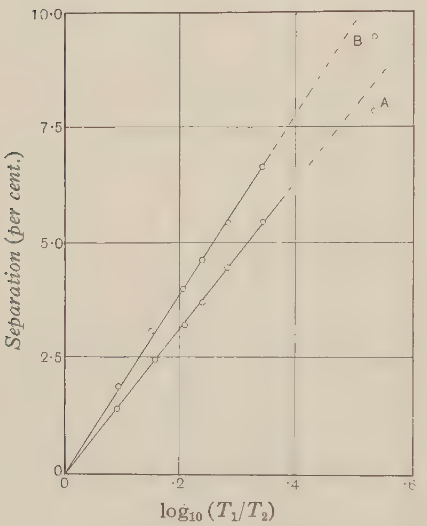

Fig. 3. Hydrogen-neon. Mixtures *A* and *B*.

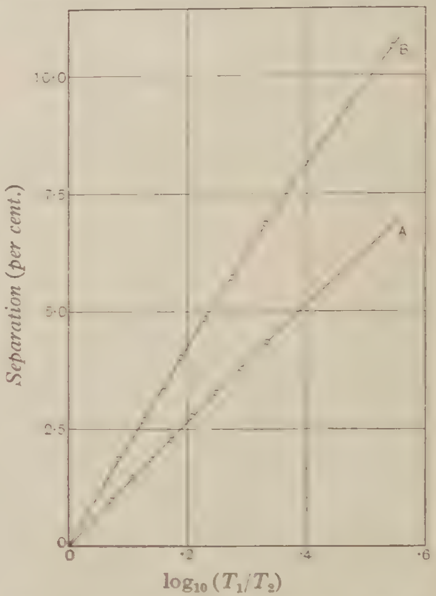

Fig. 4. Helium-argon. Mixtures *A* and *B*.

Although the falling off in this case is not great, the definite change produced by substitution of hydrogen for helium in mixtures with neon appears worthy of notice.

Using Chapman's formulae we obtain for hydrogen and neon

$$k_t = \frac{5}{2}\left\{\frac{0{\cdot}177\lambda_1 + 0{\cdot}238\lambda_2}{2{\cdot}213 + 1{\cdot}131\lambda_1/\lambda_2 + 0{\cdot}845\lambda_2/\lambda_1}\right\},$$

where λ_1 and λ_2 have the same meaning as before.

This gives to k_t, for the three mixtures *A*, *B* and *C* examined, the respective values 0·089, 0·113 and 0·123. The experimental values of k_t obtained from the curves are 0·068, 0·083, 0·090, so that R_t has the values 0·76, 0·73, 0·73. This gives a mean ordinary value of R_t of 0·74 for hydrogen-neon mixtures which is constant from 15° C. to − 145° C.

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* *Proc. Phys. Soc.* **41**, 465 (1929).

(3) *Helium and argon.* Argon at atmospheric pressure liquefies at − 186° C. but the reduction of pressure to about 20 cm. of mercury, owing to cooling which occurs in the conditions of our experiments, lowers the liquefying point sufficiently for measurements to be made with liquid air. This proved to be of considerable assistance as it enabled the complete curves for argon mixtures to be obtained without additional experimental difficulty. The results for two of the three mixtures of helium and argon are shown in figure 4. It will be seen that the proportionality of separation and $\log_{10}(T_1/T_2)$ soon begins to fail, the straight line portion of the curve being short. This is in agreement with what has been previously observed* for mixtures of hydrogen and argon for which the falling off in the value of k_t began at about − 80° C. The points obtained for helium and argon mixtures by means of cold pentane lie on a curve which can be smoothly extended to include the liquid-air point. Although the falling off in the effect is soon apparent for helium-argon mixtures, the actual bending of the curves is small. At − 190° C. the falling off in the total separation is about 8 per cent. below the proportional value, which compares with about 16 per cent. previously obtained for hydrogen and argon.

Measurements on mixtures of helium and argon with a higher range of temperature have previously been recorded†. The helium used in the earlier experiments contained a considerable amount of impurity.

Using Chapman's formulae for helium-argon mixtures we obtain

$$k_t = \frac{5}{2}\left\{\frac{0\cdot286\lambda_1 + 0\cdot244\lambda_2\,\ddagger}{2\cdot140 + 1\cdot879\lambda_1/\lambda_2 + 0\cdot448\lambda_2/\lambda_1}\right\}.$$

This gives to k_t, for the three mixtures A, B and C examined, the respective values 0·074, 0·123, 0·151. The corresponding experimental values of k_t obtained from the straight portions of the curves are 0·060, 0·094, 0·105, so that R_t has the values 0·81, 0·76, 0·69. This gives a mean ordinary value for R_t of 0·75. The present values of k_t for this pair are considerably higher than those which would be obtained from our previous measurements.

The curves for helium-argon mixtures, which are continuous throughout the temperature range, yield not only the ordinary values of k_t and R_t but also the values as they diminish at the lower temperatures. From the slope of the curve B, shown in figure 4, we obtain the approximate experimental values of k_t for different temperatures of the cold side. These values are shown in table 1. Similar tables could be prepared from the other curves.

Table 1.

T_2	0° C.	− 50° C.	− 100° C.	− 150° C.	− 190° C.
k_t experimental	0·094	0·094	0·087	0·081	0·074
R_t	0·76	0·76	0·71	0·66	0·60

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* *Proc. Phys. Soc.* **41**, 466 (1929).

† *Proc. R. S.* A, **107**, 484 (1925).

‡ Chapman and Hainsworth give $0\cdot210\lambda_2$.

Chapman's theory has shown that the value of R_t depends upon the nature of the forces operating during molecular collisions. The changing values of R_t thus provide material for the investigation of the variation of these forces with temperature.

(4) *Neon and argon.* The three curves for this pair of gases given in figure 5 show a marked falling off in the value of k_t at low temperatures. Continuous curves passing through the point given by liquid air can again be obtained. The behaviour is somewhat similar to that shown by the hydrogen-argon mixtures previously examined* at low temperatures. The separation at −190° C. is about 25 per cent. less than the proportional value. The results for this pair show the usual characteristic of argon mixtures, i.e. that the falling off in the value of k_t is soon apparent. It will also be seen that the curvature in the graphs for neon-argon is much more pronounced than in the case of helium-argon.

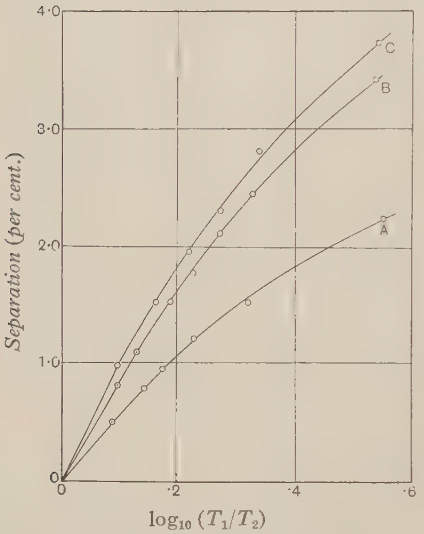

Fig. 5. Neon-argon. Mixtures *A*, *B* and *C*.

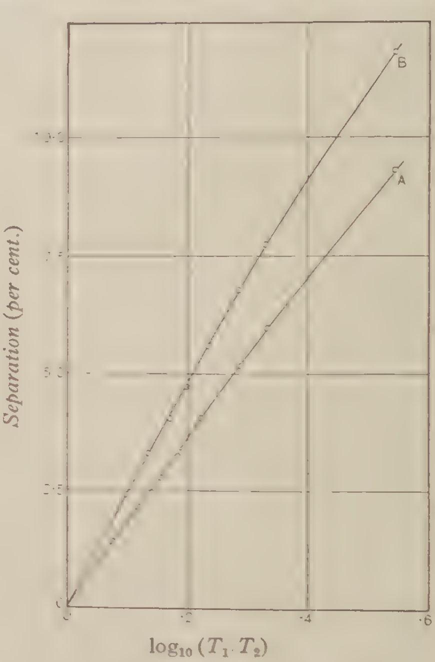

Fig. 6. Helium-nitrogen. Mixtures *A* and *B*.

Using Chapman's formulae with the numerical values inserted for neon-argon mixtures we obtain

$$k_t = \frac{5}{2}\left\{\frac{0\cdot241\lambda_1 + 0\cdot237\lambda_2}{3\cdot616 + 2\cdot368\lambda_1/\lambda_2 + 1\cdot214\lambda_2/\lambda_1}\right\}$$

This gives to k_t, for the three mixtures *A*, *B* and *C* examined, the respective values 0·043, 0·070, 0·084. The corresponding experimental values for k_t obtained from the straight portions of the curves are 0·024, 0·037, 0·043, so that R_t has the values 0·56, 0·53, 0·51. This gives a mean ordinary value for R_t of 0·53.

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* *Loc. cit.*

From the slope of the curve B shown in figure 5 we obtain the approximate value of k_t and R_t for different temperatures. These values are given in table 2.

Table 2.

T_2	0° C.	− 50° C.	− 100° C.	− 150° C.	− 190° C.
k_t experimental	0·037	0·037	0·029	0·022	0·017
R_t	0·53	0·53	0·41	0·31	0·24

(5) *Helium and nitrogen.* Two mixtures of this pair of gases were examined so that the results could be compared with those previously obtained for mixtures of hydrogen and nitrogen. The two curves in figure 6 show that the falling off in the value of k_t is much less than in the hydrogen-nitrogen mixtures. (A similar comparison has already been made of helium-argon mixtures and hydrogen-argon mixtures.) The total separation at − 190° C. is 7 per cent. less than the proportional value, compared with 20 per cent. less for the hydrogen-nitrogen mixtures. The effect of helium in the mixture is thus again well shown. The bending begins to appear in the parts of the curves obtained by means of cold pentane, i.e. before − 150° C., a feature which is characteristic of mixtures containing nitrogen. Continuous curves passing through the liquid-air point can be drawn as before.

Using Chapman's formulae with numerical values inserted for helium-nitrogen mixtures we obtain

$$k_t = \frac{5}{2}\left\{\frac{0{\cdot}322\lambda_1 + 0{\cdot}287\lambda_2}{2{\cdot}584 + 2{\cdot}137\lambda_1/\lambda_2 + 0{\cdot}554\lambda_2/\lambda_1}\right\}.$$

This gives to k_t, for the two mixtures A and B, the respective values 0·112 and 0·149. The corresponding experimental values for k_t obtained from the straight portions of the curves are 0·079 and 0·104, so that R_t has the values 0·71 and 0·70. This gives the mean ordinary value of R_t as 0·70.

From the slope of the curve A shown in figure 6 we obtain the approximate value of k_t and R_t for different temperatures. These values are shown in table 3.

Table 3.

T_2	0° C.	− 50° C.	− 100° C.	− 150° C.	− 190° C.
k_t experimental	0·079	0·079	0·077	0·071	0·064
R_t	0·71	0·71	0·69	0·63	0·57

The value of R_t at − 190° C. is thus only 20 per cent. below the value at 0° C. For hydrogen-nitrogen mixtures the corresponding variation is 50 per cent.

The results of the experiments are given in tables at the end of the paper so that they may be available if required for purposes of calculation.

§4. SUMMARY AND CONCLUSION

The experiments confirm and extend our previous work showing that there is a tendency for k_t to decrease at low temperatures. In no case have we found k_t to increase at these temperatures. A simple examination of the curves has shown that gases have characteristic effects in their behaviour in mixtures, e.g. helium in a mixture tends to make the variation of k_t small, and with argon in a mixture the change in k_t is soon apparent. A feature of the results is that for all mixtures k_t can be regarded as practically constant over a considerable range of temperature. The extent of this range depends of course upon the constituents of the mixture. This means that, within certain limits of temperature, gas-molecules during collision can be regarded as obeying an inverse-power law of repulsion, the index being constant. At lower temperatures the molecules tend to become "softer," their behaviour being less like that of rigid elastic spheres.

For the pairs of gases with weak attractive fields and low liquefying-points the value of k_t changes very little between 15° C. and − 190° C. In helium-neon mixtures the change is very small; and in hydrogen-neon mixtures there is a change below − 145° C. This is in agreement with what would be expected from the general conclusions of our previous work. The behaviour in thermal diffusion is thus related to that in viscosity* and the equations of state. It is perhaps significant that it is the hardest molecules which show the smallest variation in k_t.

Thermal diffusion may possibly provide the most direct method of studying the variations which occur in the nature of inter-molecular collisions as the temperature is reduced. A molecular model which allows for attractive as well as repulsive forces has been successful in explaining the variation of viscosity with temperature, and it may apply with equal success to thermal diffusion†. If such a model is unable to represent the observed effects at low temperatures it may be necessary to consider the possibility of changes in the actual nature of the molecular fields with temperature. Experiments on thermal diffusion at high temperatures now in progress‡ indicate the need for keeping in mind the latter interesting possibility.

Although we shall not attempt to consider in detail the theoretical aspects of the changes in k_t, it is of interest to obtain some idea of the general order of the repulsive forces arising during collision, which may be deduced from thermal diffusion data. Regarding the forces arising during collision as proportional to r^{-q}, where r is the distance between the molecules, Chapman has shown how R_t will depend upon the order q of the repulsive force. Exact values of R_t for different values of q are given in the special case§ where the ratios m_1/m_2 and σ_1/σ_2 are very large; m_1, m_2 being the respective masses and σ_1, σ_2 the respective diameters of the two molecules in the mixture. Although this is obtained for a special case it will give approximately the general information we require, shown in table 4. If we assume that repulsive forces only are operating during collision, we can consider the falling off in k_t at low temperatures as directly due to a reduction in the value of q. The effective value of q at different temperatures can thus be obtained from the different values of R_t which we have obtained for any pair of gases.

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* Cf. J. Jeans, *Dynamical Theory*, 3rd edit. 286 (1920). Table for helium.

† A theoretical investigation taking account of attractive forces is in progress. Cf. footnote by S. Chapman, *Phil. Mag.* 8, 1020 (1929).

‡ These measurements are being made by Mr A. C. R. Wakeman.

§ *Phil. Mag.* 48, 606 (1924).

Table 4. Approximate effective values of q.

Mixture	0° C.	− 50° C.	− 100° C.	− 150° C.	− 190° C.
He—Ne	18	Nearly constant			
H_2—Ne	17	Falling off below − 145° C.			
He—A	18	18	15	13	11
Ne—A	10	10	8	7	6
He—N_2	15	15	14	12	11
H_2—N_2*	10	10	8	7	6
H_2—A*	9	9	7	6·5	6†
N_2—A*	8	8	6·5	5·5	5·2†

The ordinary values of q shown in the first two columns correspond to the ordinary value of R_t for any pair. It will be seen that helium in a mixture appears to result in a high value of q. These ordinary values of q are greater than those which are at present considered to be most suitable in representing the behaviour of gases in viscosity and the equations of state. When we remember the difference in the methods of investigation, and the use of the above-mentioned approximation, the general nature of the results does not appear to be unsatisfactory. In particular, the relative values of q obtained for the different mixtures seem generally to be in agreement with such relative values as might be obtained by other means.

Further measurements are required on these and other pairs of gases. For example, the values of R_t obtained for the three helium-argon mixtures show greater differences than is usual, also the values of R_t for this pair are perhaps higher than we might expect from our knowledge of these gases. Our primary object in this work has been to continue the general study of the effect of low temperatures on thermal diffusion. With our increasing knowledge of the changes which occur in k_t the need for numerous measurements in making a general survey‡ of the phenomenon becomes more apparent. The examination of mixtures containing krypton and xenon will present additional difficulties: in this case a further reduction in the size of the diffusion apparatus will be necessary.

§ 5. ACKNOWLEDGMENT

In conclusion we desire to express our thanks to Prof. S. W. J. Smith for the help he has given by granting full facilities for this work.

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* Values obtained from results of previous paper.

† Value at − 180° C.

‡ In examining the phenomenon from different aspects some repetition of measurements is necessary. A general comparison and summary of results will be made later.

Table 5. Helium and neon.

A. 25·5 % He T_1 = 14° C.			B. 39·9 % He T_1 = 14·8° C.			C. 53·6 % He T_1 = 15·3° C.		
T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %
− 191·5	0·546	7·71	− 191·5	0·548	10·03	− 192·0	0·551	11·20
− 144·0	0·347	5·02	− 145·0	0·352	6·64	− 139·6	0·334	7·30
− 128·8	0·299	4·22	− 126·5	0·293	5·43	− 122·0	0·281	5·98
− 115·0	0·260	3·61	− 109·5	0·245	4·54	− 102·1	0·227	4·95
− 96·6	0·212	2·98	− 84·0	0·183	3·41	− 82·0	0·179	3·79
− 73·2	0·158	2·21	− 56·8	0·124	2·29	− 59·2	0·130	2·67
− 47·6	0·105	1·45	—	—	—	—	—	—

Table 6. Hydrogen and neon.

A. 27·8 % H_2 T_1 = 10·3° C.			B. 40·6 % H_2 T_1 = 12° C.			C. 49·6 % H_2 T_1 = 9° C.		
T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %
− 191·0	0·538	7·84	− 191·0	0·539	9·44	− 191·5	0·538	10·01
− 145·1	0·344	5·43	− 145·2	0·347	6·66	− 143·7	0·338	7·06
− 125·9	0·284	4·46	− 125·5	0·286	5·40	− 129·1	0·292	5·87
− 111·0	0·242	3·70	− 109·9	0·242	4·63	− 112·0	0·243	5·26
− 100·0	0·214	3·23	− 96·8	0·209	3·97	− 101·5	0·216	4·40
− 76·0	0·159	2·47	− 73·2	0·154	3·10	− 76·3	0·156	3·33
− 44·6	0·093	1·41	− 44·6	0·098	1·90	− 67·5	0·137	2·85
—	—	—	—	—	—	− 51·9	0·106	2·04

Table 7. Helium and argon.

A. 20·85 % He T_1 = 10° C.			B. 38·1 % He T_1 = 9° C.			C. 51·1 % He T_1 = 10° C.		
T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %
− 192·0	0·542	6·79	− 192·5	0·544	10·62	− 192·0	0·542	11·84
− 141·6	0·333	4·34	− 141·2	0·330	6·86	− 143·0	0·338	7·99
− 127·7	0·289	3·76	− 122·5	0·274	5·71	− 125·9	0·284	6·72
− 112·0	0·245	3·22	− 106·5	0·229	4·84	− 110·7	0·241	5·62
− 96·6	0·205	2·74	− 89·0	0·186	4·00	− 96·5	0·205	4·82
− 81·3	0·169	2·25	− 77·7	0·160	3·41	− 85·5	0·179	4·19
− 66·8	0·138	1·85	59·6	0·122	2·67	66·2	0·137	3·34
− 50·4	0·104	1·43	− 38·7	0·081	1·84	− 43·5	0·092	2·27
− 31·5	0·069	0·95	—	—	—	—	—	—

Table 8. Neon and argon.

A. 19·1 % Ne $T_1 = 10°$ C.			B. 36·1 % Ne $T_1 = 9·5°$ C.			C. 51·7 % Ne $T_1 = 10°$ C.		
T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %
− 193·6	0·552	2·22	− 191·5	0·539	3·40	− 192·0	0·543	3·71
− 137·5	0·320	1·52	− 140·3	0·328	2·44	− 143·5	0·339	2·80
− 105·0	0·226	1·21	− 121·5	0·271	2·10	− 121·3	0·271	2·30
− 83·2	0·174	0·96	− 105·0	0·226	1·78	− 101·8	0·218	1·93
− 67·5	0·139	0·79	− 88·7	0·186	1·53	− 78·7	0·163	1·52
− 39·5	0·084	0·51	− 63·7	0·130	1·10	− 46·3	0·097	0·98
—	—	—	− 46·6	0·096	0·82	—	—	—

Table 9. Helium and nitrogen.

A. 34·5 % He $T_1 = 11°$ C.			B. 53·1 % He $T_1 = 11·5°$ C.		
T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %
− 192·4	0·547	9·35	− 192·4	0·545	11·86
− 141·5	0·334	5·99	− 140·5	0·331	7·77
− 127·3	0·290	5·15	− 126·5	0·288	6·78
− 102·7	0·223	4·05	− 108·2	0·237	5·59
− 87·9	0·187	3·30	− 93·4	0·200	4·72
− 75·4	0·158	2·78	− 81·0	0·171	4·05
− 54·7	0·114	2·16	− 64·5	0·135	3·32
− 33·0	0·073	1·40	− 46·1	0·098	2·44

DISCUSSION

Prof. S. CHAPMAN. I should like to congratulate the authors on this very valuable and comprehensive addition to our knowledge of the facts about thermal diffusion. Dr Ibbs, with the assistance of his collaborators, has made himself the leading experimental expert on thermal diffusion, and in this and another recent paper on the phenomenon as it occurs at low temperatures has gone far to remove the apparent discrepancies that had arisen between the work of various experimenters on the subject. I think the discussion in this paper is as complete and instructive as can be given without entering into complicated mathematical aspects that are not yet fully worked out. I hope and anticipate that before long the theory may be further developed to the extent necessary for the fuller discussion of the many results now accumulated by Dr Ibbs and his colleagues.

Prof. A. O. RANKINE. I would like to ask whether the effect of the partial liquefaction of one of the components of the mixture is to be regarded as an extreme case of thermal diffusion. Clearly, if the cold bath is cold enough the less volatile of the constituents will liquefy in part first, thereby reducing largely its concentration in the warmer chamber. Is this separation true thermal diffusion? On this point it should perhaps be noticed that the separation by differential liquefaction may be either of the same sign as that to be expected from the relative masses of the molecules, or of the opposite sign. Take water vapour and oxygen, for example. The water-vapour molecule is of smaller mass than the oxygen molecule, yet its liquefying point is much higher. Presumably in this case the two effects would be oppositely directed.

Prof. Sir A. S. EDDINGTON said that according to table I of the paper the molecules get softer and softer as the temperature falls. There must come a point, therefore, where the sign of the thermal-diffusion effect is reversed.

Prof. S. CHAPMAN. The reversal of the thermal-diffusion effect does not occur till the force-index falls below 5, a state of things of which there is no sign in these experiments. With reference to Prof. Rankine's remarks about liquefaction, one would expect that the gas composed of the larger and heavier molecules, which go preferentially to the cold end, would be the first to liquefy; hence it seems unlikely, though perhaps not impossible, that the two effects should work in the same direction.

AUTHORS' reply. Although we have not yet observed a case of the reversal of the thermal-diffusion effect as mentioned by Sir Arthur Eddington, we have kept in mind this interesting possibility during the progress of the work.

The question raised by Prof. Rankine regarding the possibility of partial liquefaction of one of the gases in the cold vessel is an important one, although any separation as the result of such condensation should not be considered as due to thermal diffusion. We have already considered this possibility with some care in our previous paper*, and concluded that the simple gas conditions could be regarded as holding throughout the experiments. At low temperatures we observe a decrease in the value of k_t, which means that the separation obtained is less than normal, whereas any partial condensation of the heavier gas would tend to increase the separation and make the apparent value of k_t greater than its true value. The example of a mixture of water vapour and oxygen, mentioned by Prof. Rankine, appears to provide a rather special case in which the lighter molecules would be the first to condense.

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* *Proc. Phys. Soc.* **41**, 468 (1929).

FURTHER EXPERIMENTS ON MAGNETOSTRICTIVE OSCILLATORS AT RADIO-FREQUENCIES

By J. H. VINCENT, M.A., D.Sc., F.Inst.P.

Received November 14, 1930. *Read and discussed January* 16, 1931.

ABSTRACT. Work on magnetostrictive oscillators at radio-frequency is continued from a previous paper. It is shown that the coil surrounding the bar can be in either branch of a simple tuned anode circuit. When the bar-coil is in the inductive branch the circuit may be operated as a series or parallel arrangement; in the latter case the direct plate current does not pass through the inductive branch of the fly-wheel circuit. The variation in either the anode or the grid current can be used to indicate resonance. Comparative experiments with corronil, nickel and glowray suggest that glowray is the most suitable of these materials for high-frequency oscillators. The experiments are carried on to the case of a glowray oscillator 1·9 mm. long which has a frequency of 1280 kc./sec. and weighs less than a fiftieth of a gramme.

§ 1. INTRODUCTION

This paper is a continuation of one of similar title* and records some further experiments on small magnetostrictive oscillators.

§ 2. THE OSCILLATOR AND COIL IN EITHER BRANCH OF THE TUNED CIRCUIT

In the discussion on the former paper, Dr D. Owen suggested that it would seem preferable to include the coil surrounding the oscillator in the condenser branch of the tuned circuit rather than in the inductance side, since in the latter case the variation of the plate current constitutes an unnecessary variation in the polarization of the oscillating bar. In my printed reply I described some work which had been done on this point but too recently to be included in the body of the paper. This showed that the coil surrounding the oscillator could be included in either branch and afforded an example in which the stability had a greater range with this coil in the condenser branch. Figure 1 gives the arrangement used in these experiments. The pivot P of the two-way switch is joined to the negative terminal of the high-tension battery and the ends of the oscillator coil L_3 are connected to the other two electrodes of the switch. By changing the switch arm from PR to PQ the experimenter could transfer the coil L_3 from the inductive branch to the capacitative branch of the oscillatory circuit. In the position PR the coil carries the steady plate current; in the position PQ the coil L_3 is traversed by alternating current only. It was found that the mutual inductance between L_1 and L_2 had to be increased in order to maintain the circuit in oscillation when the position was PQ.

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* *Proc. Phys. Soc.* **41**, 476–86 (1929).

§ 3. BAR COIL IN THE INDUCTIVE BRANCH

The assemblage shown in figure 2 was set up in order to investigate the effect of the steady anode current when the oscillator coil is still in the inductive side of the fly-wheel circuit. It allows the plate current to traverse L_3 when the switch arm is in the position *AB*. This will be called the series arrangement. When the switch is in the position *AC* the anode current returns to *A* through R_2, L_3 being traversed by alternating current only. This will be called the parallel arrangement. The direct-current resistances of these alternative paths were made identical by the resistances R_1 and R_2. In either case the direct component of the current supplied by the high-tension battery was measured by A_m.

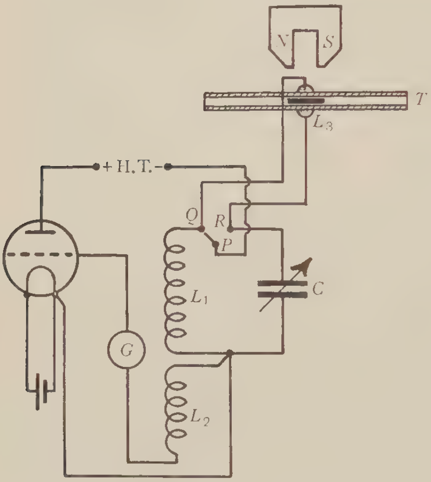

Fig. 1.

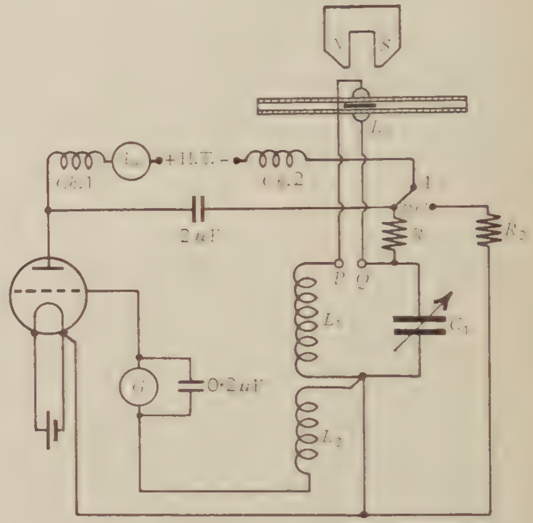

Fig. 2.

In figure 2, *P* and *Q* are small mercury cups into which the wires leading to L_3 dip. By exchanging the position of these wires the direction of the steady anode current could be changed when the switch was in the position *AB*. The coil L_3 has 400 turns of No. 42 enamelled copper wire (diameter 0·1 mm.) wound on a thin-walled glass tube in which was placed the glowray oscillator, 6 mm. in length, used in the previous paper. L_1 is a close-wound single-layer coil of 60 turns of No. 24 d.w.s. copper (diameter 0·56 mm.) on a cylindrical former 9 cm. in diameter; the grid coil L_2 is similarly constructed of 75 turns of No. 26 d.w.s. copper (diameter 0·46 mm.) on a former 8·2 cm. in diameter. L_1 and L_2 were mounted coaxially with the distance between their centres kept at 6·5 cm. during the experiments described below. The valve is an R.C. 210 B.T.H. and was operated with 90 volts on the plate and 1·9 volts on the filament, with no grid bias.

§ 4. ANODE CURRENT PASSING THROUGH L_3

Figure 3 gives a set of readings of the grid galvanometer G, the anode galvanometer A_m and the condenser of the absorption wavemeter, plotted against the values of C_1 as abscissae, for the series arrangement in which the steady current through L_3 assists the polarization due to the permanent magnet NS.

It is seen from figure 3 that with this disposition of apparatus the indications of either the grid or the anode galvanometer can be used to indicate resonance. Oscillation-hysteresis is indicated by arrows as in the previous paper.

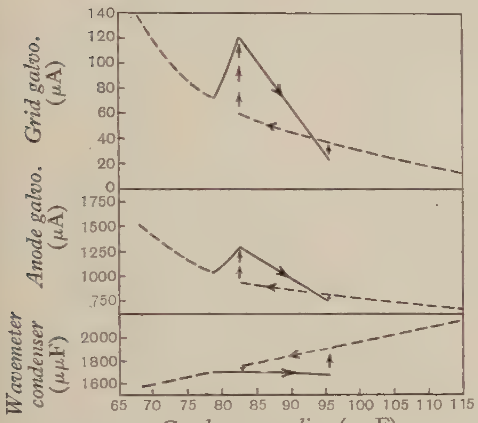

Fig. 3. Series arrangement.

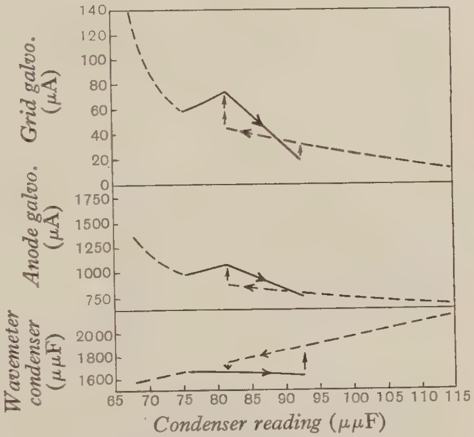

Fig. 4. Series arrangement with bar coil leads reversed.

If now the leads to P and Q from L_3 be interchanged, all else being kept the same, the results (shown in figure 4) are of the same general character; in particular the range of variation of the condenser C_1 over which the set is stabilized by the magnetostrictive oscillator is sensibly unaltered. The peaks of the curves of the galvanometer readings are, however, much less pronounced when the anode current is sent through L_3 in the direction to oppose the polarization due to the magnet.

§ 5. ANODE CURRENT NOT PASSING THROUGH L_3

When the leads to L_3 are in their original position so as to assist the polarization in the series arrangement, change of the switch from the AB to the AC position, which alters the mode of operation of the set from series to parallel working, produces no appreciable change in the curves. This is shown in figure 5, taken with the switch in the AC position and the leads to L_3 in the same position as for figure 3.

It might be expected that on reversal of the leads to L_3 no effect on the curves would be now found. But figure 6, taken after this reversal, shows that the current peaks are depressed very similarly to the case in which the steady anode current flows through L_3. This shows that with the particular conditions of these experiments the reversal of phase of the oscillatory current in L_3 with respect to that in L_1 is not immaterial. This is probably due to a lack of symmetry in the wave-form of the oscillating current.

The wave-length corresponding to 1700 $\mu\mu$F on figures 3, 4, 5 and 6 is 719 m. and the frequency 417 kc./sec.

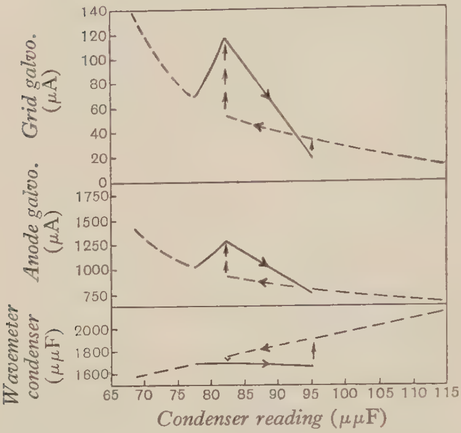

Fig. 5. Parallel arrangement.

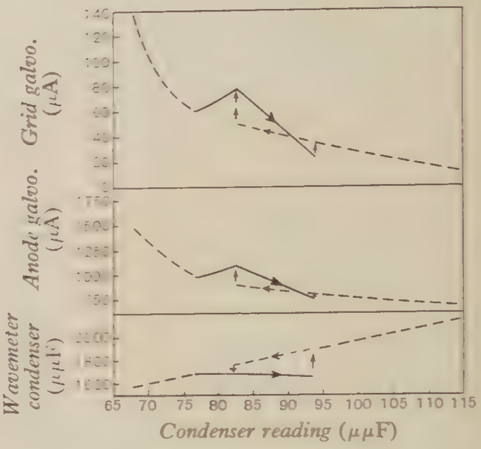

Fig. 6. Parallel arrangement, bar coil leads reversed.

From these experiments we see that the indicating galvanometer may be in the anode or grid circuit and the bar coil L_3 may be in the inductive or capacitative branches of the fly-wheel circuit.

§ 6. EXPERIMENTS WITH CORRONIL AND NICKEL

Replicas of the 6 mm. glowray bar were made in corronil and nickel. Corronil is an alloy, similar to monel, of the approximate percentage-composition 71 Ni, 25 Cu, 4 Mn. Pierce* found monel to be a very powerful oscillator. The bars of nickel and glowray behaved very similarly but the effects produced by corronil were very much more pronounced. Corronil, however, was not found to be a suitable material for high-frequency oscillators as its behaviour shows a marked instability due to change of temperature. In order to obtain reproducible curves it is necessary to use very small oscillatory fields. This effect shows itself as follows.

When the current in the grid galvanometer (and thus when the oscillatory current) is high, the deflection increases with time. If now the capacity is changed so as to reduce the deflection to a small value the deflection slowly decreases to a steady value. This instability affects all parts of the grid-galvanometer capacity curve; the top of the resonance peak, however, is less affected than the remainder of the graph and soon attains a steady value. These effects are probably due to the magnetic permeability of corronil changing rapidly with temperature. The frequency corresponding to the peak value does not change noticeably so that these drifts are due to magnetic rather than mechanical changes. Cooling the oscillator by allowing a few drops of ether to evaporate from the bar coil and glass tube lowers the deflection of the galvanometer. If the cooling be carried lower than the dew point, the deposit of water acts as a clamp on the oscillator and the peak disappears from the curves but is restored when the tube and oscillator are dried. Glowray is much freer from these thermal drifts than corronil, as also is nickel, but nickel does not possess the same stabilizing properties as glowray. Pierce[*] remarks on the lack of stabilizing power of nickel; by comparison of the 6 mm. bars of glowray and nickel in the same apparatus, the range of variation of capacity over which the frequency was stabilized was found to be about 1·7 times as great for glowray as for nickel.

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* *Proc. Amer. Acad. of Arts and Science*, **63**, 1–47 (1928).

§7. CIRCUITS WHICH OSCILLATE ONLY AT THE CONTROLLED FREQUENCY

Since corronil is such a vigorous oscillator it is possible to repeat at these frequencies (378 kc./sec.) an experiment already described with longer bars. This is the reduction of the grid coupling to such an extent that the passage through resonance is shown by a peak which is quite isolated, being flanked in either capacity-direction by a region in which the set does not oscillate[†]. This experiment is illustrated by figure 7 which shows a set of readings of the grid galvanometer (a unipivot of resistance 43 ohms) plotted against the capacity C, the apparatus being arranged as in figure 1. A large condenser not shown in this figure was connected across the plate battery. The valve was R.C. 210 B.T.H. operated at 100 volts with no grid bias. The 400-turn coil surrounding the corronil bar (6 mm. in length and 2·135 mm. in diameter) was in the inductive branch of the fly-wheel circuit. L_1

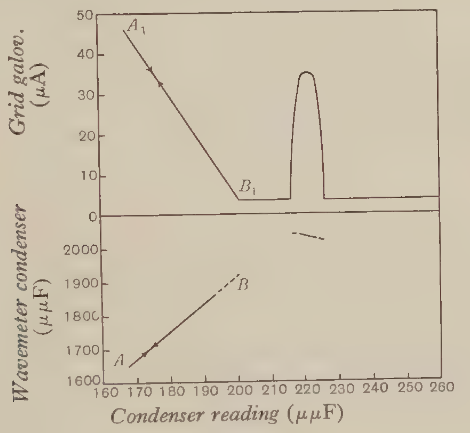

Fig. 7.

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[*] *Loc. cit.*

[†] *Electrician*, **102**, 11 (1929).

was a single-layer coil of 40 turns of No. 24 d.w.s. (diameter 0·56 mm.) closely wound on a former 9·2 cm. in diameter. The grid coil L_2 was a similar coil of 75 turns of No. 26 d.w.s. (diameter 0·46 mm.) on a former 8·1 cm. in diameter. The mutual induction between these coils was reduced by trial until the resonance peak was isolated.

The upper part of figure 7 shows the way in which the steady grid-current changes as the capacity is decreased from a high value. The set is not oscillating at first but begins to oscillate when the capacity is reduced to 226 $\mu\mu$F; the deflection reaches a maximum at 222 $\mu\mu$F and returns to its former low value at 216 $\mu\mu$F, when the set is again quiescent. Throughout this range the set is approximately stabilized, the frequency falling slightly as the capacity is decreased. This is shown on the lower portion of the figure in which the ordinates are the readings of the condenser in the wavemeter circuit, the condenser value 2040 corresponding to a frequency 378 kc./sec. On further decrease of the capacity the set begins to oscillate again at about 200 $\mu\mu$F and now the stabilization is entirely absent, as is indicated by the line BA in the figure. In the part B_1A_1 of the grid-current curve the effects of change of temperature on the magnetic properties of the oscillator are pronounced, the recorded deflections depending on the time occupied at each setting of the condenser. This thermal drift does not sensibly affect the frequency, so that, although the curve A_1B_1 does not repeat itself, AB is quite definite. Apart from this thermal effect, oscillation-hysteresis is absent and the isolated peak is found in the same position when the capacity changes are made in the reverse direction.

Experiments of this type are easily performed with a corronil oscillator, and with different valves and bar coils. For instance, with this particular oscillator it is quite easy to obtain the isolated peaks even with a bar coil of only 50 turns, the bar being located in either branch of the fly-wheel circuit.

§ 8. EFFECT OF DECREASING THE NUMBER OF TURNS IN THE COIL SURROUNDING THE OSCILLATOR

When the number of turns surrounding a nickel oscillator is reduced, the peak on the grid-current/capacity curve is reduced and the falling portion of the curve becomes steeper as capacity is increased, while indications of oscillation hysteresis decrease until they become negligible. At the same time the range of stabilization becomes less until one finally gets a mere discontinuity, coinciding with the crevasse, in the frequency curve. This is illustrated by two pairs of curves, figures 8 *a* and 8 *b*. In figure 8 *a* a 6-mm. nickel oscillator is surrounded by a 400-turn coil, while in figure 8 *b* the coil has only 50 turns. The wavemeter capacity of 1650 $\mu\mu$F corresponds to a frequency of 420 kc. The two pairs of curves were drawn under nearly the same conditions, the mutual inductance between L_1 and L_2 being less in the case of figure 8 *b*. Figure 8 *a* shows that the stabilizing power of the nickel oscillator is limited to the falling part of the peak on the grid-current curve, and does not include the rising part of the peak on its low-capacity side as in the case of glowray oscillators. (See figures 2 and 3 of the former paper.) Curves similar to those of figure 8 *b* could be obtained with different valves by proper adjustment of the coupling between the coils L_1 and L_2.

If a pair of telephones are inserted in the plate circuit under the conditions for producing a curve like that of figure 8 *b*, then if the capacity be decreased regularly a slight, ill-defined click is heard on passage through point *A* immediately to the left of the bottom of the crevasse. No sound was produced in the telephones when the curve was traversed in a reverse direction.

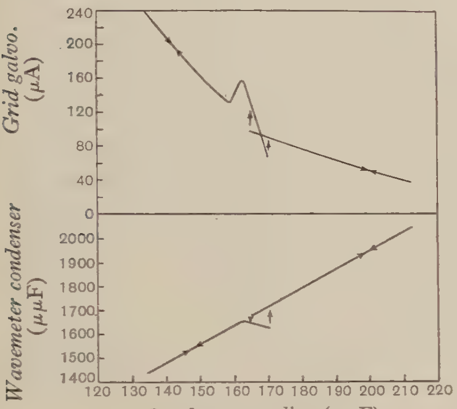

Fig. 8*a*.

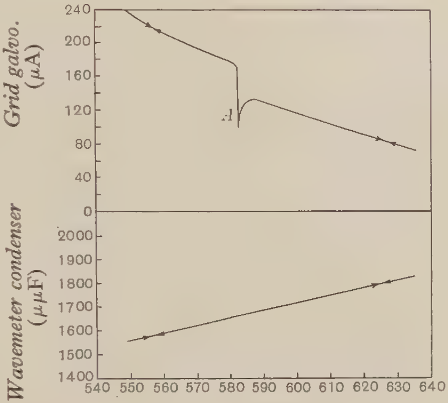

Fig. 8*b*.

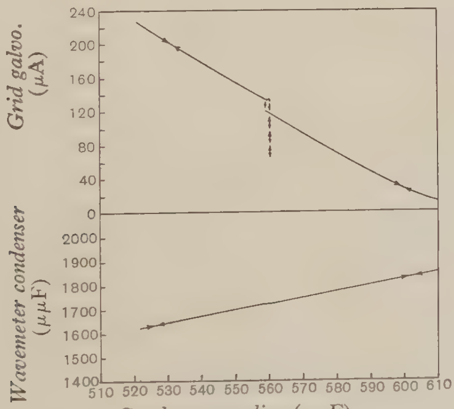

Fig. 9.

When a similar glowray bar is substituted for the nickel bar in figure 8 *b* marked oscillation-hysteresis is evident. This is illustrated in figure 9. When the curve is traversed from left to right a distinct peak, followed by a deep crevasse, is shown, while on the return path from right to left the deflection suddenly increases when the capacity has fallen to a certain value, and the crevasse and peak are missed entirely.

The possibility of obtaining a sharp crevasse in the grid-current capacity curve with oscillators about 6 mm. in length suggested a further trial of shorter oscillators. The 4·5-mm. glowray oscillator (used in the former paper) was tried in a bar coil of 50 turns with an R.C. 210 B.T.H. valve. This gave a sharp crevasse when the curve was traced in the direction of increasing capacity, the measured frequency at the left-hand side of the top of the crevasse being 540 kc. sec. This value is identical, within the limits of accuracy of measurement, with the value obtained previously under very different circumstances, the valve, bar coil and polarizing field all being different.

When the curve was traced in the opposite direction a very small dip occurred in the curve at the value of the condenser corresponding to the right-hand side of the crevasse and was followed closely, on a slight decrease of the capacity, by a sudden rise in the grid current. The crevasse was thus apparently eliminated from the trace taken with decreasing values of capacity. This small dip in the curve, which precedes a sudden rise on decrease of the capacity, is a very usual feature of such curves, but is more noticeable when experiments are made at much lower frequencies. At low frequencies, however, the dip is not changed in character even if the changes in capacity are made exceedingly slowly. With the 4·5-mm. glowray bar, on the other hand, the dip is transformed into a crevasse if the trace from right to left is made with very small decrements of capacity.

§ 9. EXTENSION OF THE EXPERIMENTS TO HIGHER FREQUENCIES

These results suggested that if one varied the capacity very gradually it would be possible to detect the crevasse with much shorter oscillators, even when the number of turns in the bar coil was diminished. This was found to be the case and no difficulty was experienced in carrying the experiments down by easy stages to the case of an approximate cylinder of glowray about 1·9 mm. long and about 1·2 mm. in diameter: this weighs less than a fiftieth of a gram. The frequency of the bar was 1280 kc./sec., corresponding to a wave-length of 234 m. There does not seem to be any reason why such experiments could not be extended to much higher frequencies.

As the bars were made successively shorter, oscillation-hysteresis and stabilization of frequency decreased. It was found necessary also to increase the polarizing magnetic field and to work with oscillating currents of smaller amplitude.

It was possible to detect resonance with these short bars when the bar coil was in either branch of the fly-wheel circuit and with circuits differing very widely from each other. The frequency corresponding to resonance was, to a close approximation, constant throughout such changes.

In the case of the bar just mentioned, when it was surrounded by a coil of 25 turns of No. 42 enamelled copper wire (diameter 0·1 mm.) in the inductive branch of the circuit, the whole irregularity in the grid-current condenser graph (including the crevasse and the slight departures from uniformity outside it on both sides) occurred within a range of 2 $\mu\mu$F in a total capacity of 575 $\mu\mu$F.

§ 10. ACKNOWLEDGMENTS

I have great pleasure in thanking my students, Messrs M. S. Hoban and G. E. Lloyd, for assistance in taking readings and Mr Lloyd for drawing the diagrams.

DISCUSSION

Mr S. Butterworth. I wish to congratulate Dr Vincent on having succeeded in maintaining a magnetic rod in oscillation at a frequency of more than $10^6 \sim$. The practical value of these mechanical oscillators is that they serve to supply accurate standards of frequency in a compact and convenient form. The phenomena shown can be imitated with purely electrical circuits, but in order to obtain sharp and precise indications these circuits must have very small damping which can be achieved only with bulky coils. The suitable circuit consists of a resistance and conductance shunted by a resonant circuit consisting of resistance, capacity and inductance.

THE EQUIVALENT CIRCUIT OF THE MAGNETOSTRICTION OSCILLATOR

By S. BUTTERWORTH, M.Sc. and F. D. SMITH, M.Sc., A.M.I.E.E.

Received November 1, 1930. *Read and discussed January* 1, 1931.

ABSTRACT. The Joule effect in magnetostriction is defined by the relation $S = \lambda B$, where S is the stress produced by a small increment of induction B of an existing induction B_0; and the converse effect by the relation $H = \kappa\xi$, where H is the magnetic field produced by a small strain ξ. Conservation of energy requires that $\kappa = 4\pi\lambda$. These equations are used in investigating the motion of a magnetostriction oscillator consisting of a closed circular ring inside a toroid, eddy currents and hysteresis being taken into account. It is shown that the oscillator can be represented by an equivalent electrical circuit comprising a pair of parallel impedances Z_c, Z_m, in series with an impedance Z_l, where Z_l represents leakage impedance and Z_c impedance due to core flux in the absence of motion, while Z_m is a resonant shunt to Z_c and represents the effect of the motion. Expressions for the elements of the circuit in terms of the fundamental constants of the material are given. The circle-diagram of impedances is deduced, and the modifying effects of eddy currents and hysteresis are investigated. Some simple geometrical relations between the vectors in the diagram are derived. It is shown that the size of the circle diagram for solid material having a large value of λ depends mainly upon permeability and resistivity and only slightly upon λ, and that a high degree of resonance may be associated with poor magnetostriction quality; the circle-diagram may, in fact, lead to erroneous conclusions unless interpreted in the light of an adequate theory. An experimental investigation of the resonant radial vibrations of solid and laminated nickel rings verifies the theoretical deductions and leads to numerical values of λ and κ in agreement with values deduced from the work of Masumoto and Nara and of McKeehan and Cioffi. For nickel in the annealed state, $\lambda = 1{\cdot}76 \times 10^4$ and $\kappa = 22{\cdot}1 \times 10^4$ at a point on the curve corresponding to $H_0 = 14{\cdot}5$ gauss.

§ 1. INTRODUCTION

The work hitherto published on the magnetostriction oscillators, proposed by Pierce$^{(1)}$ as standards of frequency, has been mainly of an experimental nature. The only theory available is based on initial assumptions which avoid the complications arising from eddy currents and hysteresis. With solid magnetostrictive materials, eddy currents are important since their effect is to confine the alternating magnetic flux to the surface of the vibrator, and it is desirable to develop a theory which takes this into account.

In common with other mechanical vibrators maintained in vibration by electrical means, the magnetostriction oscillator can be represented by an equivalent electrical circuit. When this equivalent circuit is known, the magnetostriction oscillator with the associated valve circuits can be considered as a single electrical circuit from which the conditions for the maintenance of mechanical vibrations and the optimum values for the components of the valve circuit can be found by standard methods. Formulae for calculating the elements of the equivalent circuit from the fundamental properties of the magnetostriction oscillator are derived in the course of the theoretical analysis; these formulae show the effect of any changes in the material, dimensions or coil of a magnetostriction oscillator and are of some assistance in arriving at the best oscillator for any specific purpose. It is to be noted, however, that the magnetostriction oscillator should always be considered in association with the external circuit maintaining it in vibration. The problem of finding the best oscillator to use under a given set of circumstances cannot be solved by mere substitution of rods or rings of various metals and sizes in a single circuit, as the circuit may be nearer optimum conditions for one rod than for another. Considerations of this kind may account for a statement by Pierce(1) that nickel is not the best material to use when employing magnetostrictive systems to control the frequency of a valve oscillator. Nickel shows a large magnetostriction effect and it is possible that a smaller effect was required to satisfy optimum conditions in his circuits.

A sound fundamental theory is therefore essential if the best results are to be obtained without an excessive number of experiments. This paper attempts to set out this theory and to show how it may be used to derive the fundamental properties of magnetostrictive materials from motional-impedance measurements.

§ 2. FUNDAMENTAL PRINCIPLES

If a magnetized material undergoes a small cyclic change B of induction, an increment in stress S is produced which is proportional to B, so that we may write

B
S

$$S = \lambda B \qquad \text{......(1).}$$

The factor λ may be positive or negative and its value depends upon the steady induction about which the small change B takes place.

λ

Conversely, if the magnetic material undergoes a small cyclic fractional extension ξ, a small magnetizing force H is induced which is proportional to ξ so that

ξ, H

$$H = \kappa\xi \qquad \text{......(2).}$$

κ

If we ignore the complication of hysteresis it may be shown from the principles of the conservation of energy that the factors λ and κ are not independent, and if we use the electromagnetic system of units we have the relation

$$\kappa = 4\pi\lambda \qquad \text{......(3).}$$

In this system it is interesting to notice also that the square of λ (or κ) has the dimensions of a stress.

§ 3. EQUATION OF MOTION OF MAGNETOSTRICTIVE RING

Consider a ring of magnetostrictive material such as that shown in figure 1.
r, b, d The ring has mean radius r and rectangular cross-section b, d. If the material has
E, σ Young's modulus E and density σ, then for a peripheral strain ξ the increase in
x mean radius is x, or $r\xi$, so that the potential energy due to the strain is V, where

γ

$$V = \tfrac{1}{2}E\,(x^2/r^2)\,.\,2\pi rbd = \tfrac{1}{2}\gamma x^2, \text{ say} \qquad \text{......(4)},$$

in which

$$\gamma = 2\pi bdE/r \qquad \text{......(5)}.$$

T Also for varying strains the ring has kinetic energy T given by

$$T = \tfrac{1}{2}\sigma \times 2\pi rbd\dot{x}^2 = \tfrac{1}{2}\alpha\dot{x}^2 \qquad \text{......(6)},$$

α in which

$$\alpha = 2\pi rbd\sigma \qquad \text{......(7)}.$$

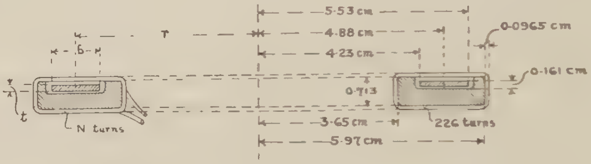

Fig. 1.

F If, further, we assume the dissipation function F to be given by

$$F = \beta\dot{x}^2 \qquad \text{......(8)},$$

the equation of motion follows from the usual dynamical theory and is

$$\alpha\ddot{x} + \beta\dot{x} + \gamma x = \psi \qquad \text{......(9)},$$

ψ in which ψ is the force producing the strain.

E, β The modulus E and the damping coefficient β must be carefully defined. Any straining of magnetostrictive material produces magnetic force and, if this strain varies, eddy currents which absorb energy from the motion and produce damping are set up in the material. The factor β in equation (9) is not supposed to include this damping, which will be taken into account as part of the force ψ in the later equations. Also the stress produced by the induction caused by the above magnetic force is proportional to the strain and so may legitimately be regarded as producing a modification of Young's modulus. It would be so included if the modulus in question were measured with the material under magnetization, and in the case of oscillators its value would depend not only upon the state of magnetization but also on the rate of alternation of the strains. The Young's modulus to be used in finding the γ factor of equation (9), however, is supposed to exclude this variable factor. Its effect will be included as part of the force ψ in the later equations.

For the present applications the force ψ is due to the total flux across the section of the ring, which is produced primarily by a variable current flowing in the surrounding toroidal coil. Owing to eddy-current effects the induction will vary in magnitude and phase across the ring-section and, if ϕ be its integral value, the integral stress will be $\lambda\phi$ by (1). This stress produces a radial component of force $\lambda\phi . \delta\theta$ on an element subtending an angle $\delta\theta$ at the centre of the ring. Since each element is subject to a similar radial force, the value of ψ is given by

$\delta\theta$

$$\psi = 2\pi\lambda\phi \qquad \text{......(10).}$$

Using this value of ψ in equation (9) and assuming alternating flux, we now have for the equation of motion

$$(\gamma - \alpha\omega^2 + j\omega\beta)\, x = 2\pi\lambda\phi \qquad \text{......(11),}$$

in which x and ϕ now represent rotating vectors having pulsatance ω, and j is the operator denoting anti-clockwise rotation through a right angle on the vector diagram.

$x, \phi, \omega,$

For the purposes of abbreviation we may write (11) in the form

$$\nu x = 2\pi\lambda\phi \qquad \text{......(12),}$$

in which

$$\nu = \gamma - \alpha\omega^2 + j\omega\beta \qquad \text{......(13).}$$

ν

§ 4. RELATION BETWEEN INTEGRAL FLUX, DISPLACEMENT AND CURRENT

If the ring is surrounded by a toroidal coil having N turns and carrying a current $I\epsilon^{j\omega t}$, the magnetizing force due to this current has amplitude $2NI/r$. In addition, there is the magnetizing force due to the strain which, by equation (2), is $\kappa x/r$. Hence the total amplitude of the magnetizing force is given by

N
I, t

$$H = (2NI + \kappa x)/r \qquad \text{......(14).}$$

For very low frequencies the integral flux produced by H may be written

$$\phi = \mu H b d \epsilon^{-j\eta} \qquad \text{......(15),}$$

in which μ is the reversible permeability and η the angle of lag, due to hysteresis, at the working point on the magnetization curve of the ring.

μ, η

The formula requires modification when eddy currents are appreciable and then we must replace (15) by

$$\phi = \mu H b d \epsilon^{-j\eta} \chi \qquad \text{......(16),}$$

in which χ is the eddy-current vector defined by its amplitude χ_0 and lagging phase ζ so that

χ, χ_0
ζ

$$\phi = \mu H b d \chi_0 \epsilon^{-j(\eta+\zeta)} \qquad \text{......(17).}$$

The in-phase and quadrature components of χ will be denoted by χ_R and χ_I which are related to χ_0 and ζ by

χ_R, χ_I

$$\chi_R = \chi_0 \cos\zeta, \quad \chi_I = \chi_0 \sin\zeta \qquad \text{......(18).}$$

Although χ is obviously a "dimensionless" quantity, yet it depends upon the linear scale of the ring section since one of the lengths involved in the determination of χ is the wave-length of propagation of electromagnetic disturbances in the material of the ring, and the linear size enters as a multiple of this wave-length.

If the ring section is such that d/b is small, then we can obtain an approximate value of χ from the ordinary theory of penetration into an infinite plane sheet of alternating magnetic forces. This theory gives

$$\chi = \theta^{-1} \tanh \theta \qquad \text{......(19)},$$

θ, f, ρ where $\theta = \pi d\,(2j\mu f/\rho)^{\frac{1}{2}}$, f being the frequency and ρ the resistivity of the material. L Since $(\rho/\mu f)^{\frac{1}{2}}$ is the wave-length, L, appropriate to electromagnetic waves in the material, we may denote the dimensionless nature of θ (and therefore of χ) by writing

$$\theta = \pi . (2j)^{\frac{1}{2}}\, d/L \qquad \text{......(20)}.$$

χ_0, ζ, χ_R, and χ_I are thus real functions of the real variable z, where

z
$$z = \sqrt{2} . \pi d/L.$$

Table 1, which gives these functions, has been obtained from Kennelly's *Tables of Complex Hyperbolic and Circular Functions*⁽⁹⁾.

Table 1. Table for computation of eddy-current factors.

z	χ_0	ζ degrees	χ_R	χ_I
0	1·000	0·0	1·000	0·000
0·2	1·000	0·8	1·000	0·014
0·4	0·998	3·0	0·997	0·052
0·6	0·990	6·8	0·983	0·117
0·8	0·970	11·9	0·949	0·200
1·0	0·931	18·0	0·885	0·288
1·2	0·871	24·4	0·793	0·360
1·4	0·796	30·5	0·686	0·404
1·6	0·713	35·8	0·578	0·417
1·8	0·632	39·9	0·485	0·405
2·0	0·560	42·9	0·410	0·381
2·2	0·497	44·8	0·353	0·350
2·4	0·445	46·0	0·309	0·320
2·6	0·402	46·5	0·277	0·291
2·8	0·367	46·6	0·252	0·267
3·0	0·338	46·5	0·233	0·245

For large values of z, χ_0 settles down to the value $1/z$, ζ becomes 45° and $\chi_R = \chi_I = 0{\cdot}707/z$.

The above formulae and table 1 apply not only to a single ring of solid material but also to a composite ring built up of a number of flat rings. d then refers to the thickness of each ring and bd in (17) must be replaced by the total metallic section of the composite ring.

Using (14) in (17) we now have for the relation between integral flux, current and displacement

$$\phi = \mu bd\,(2NI + \kappa x)\, \chi_0 \epsilon^{-j(\eta+\zeta)}/r \qquad \text{......(21)}.$$

§ 5. CONTRIBUTION OF RING TO IMPEDANCE OF TOROIDAL COIL

The alternations of ϕ induce in the toroidal coil a back electromotive force v of amount $j\omega N\phi$, and if x is eliminated between the two equations (12) and (21), we find, on putting $j\omega N\phi = v$, that the relation between I and v may be written v

$$I = v\,(1/Z_c + 1/Z_m) \qquad \text{......(22)},$$

in which

$$Z_c = 2j\omega N^2\,\mu b d \chi_0\,\epsilon^{-j(\eta+\zeta)}/r,$$ Z_c

$$Z_m = -\,j\omega N^2 \nu/\pi\lambda\kappa \qquad \text{......(23)}.$$ Z_m

Thus the ring behaves as if it consisted of two impedances Z_c and Z_m in parallel.

The impedance Z_c is independent of the magnetostrictive and motional properties of the material. It is, in fact, the core impedance that would be obtained if the ring could be rigidly clamped. The factor $2N^2\mu bd/r$ is the core inductance at very low frequencies when hysteresis is neglected; we may denote it by L_0. Z_c may, then, be analysed into a resistance component R_c and a reactance component ωL_c. These components are given by the formulae L_0 R_c, L_c

$$R_c = \omega L_0 \chi_0 \sin(\eta + \zeta), \quad L_c = L_0 \chi_0 \cos(\eta + \zeta) \qquad \text{......(24)}.$$

The impedance Z_m is independent of eddy currents and hysteresis and represents the reaction of the ring motion on the toroid. Using the value of ν in equation (13) we find that

$$Z_m = \frac{\omega^2}{\omega_0^2}\frac{N^2\gamma}{\pi\lambda\kappa}\left\{\frac{\beta}{\alpha} + j\omega\left(1 - \frac{\omega_0^2}{\omega^2}\right)\right\} \qquad \text{......(25)},$$

in which $\gamma/\alpha = \omega_0^2$, and ω_0 is to be interpreted as the free pulsatance of the ring. We may write (25) thus: ω_0

$$Z_m = R_m + j\,(\omega L_m - 1/\omega C_m) \qquad \text{......(26)},$$

provided that

$$L_m = N^2\gamma\omega^2/\pi\lambda\kappa\,\omega_0^2, \quad R_m/L_m = \beta/\alpha, \quad L_m C_m = 1/\omega_0^2 \qquad \text{......(27)}.$$ L_m, R_m,

It will be noticed that L_m is proportional to the square of the pulsatance ω, but as in all practical cases the value of Z_m is very high except when the pulsatance is in the neighbourhood of ω_0, we may without much error regard ω as equal to ω_0 in (27), and then with the help of (5) we get

$$L_m = N^2\gamma/\pi\lambda\kappa = 2N^2 bd\,E/\lambda\kappa r = L_0 E/\lambda\kappa\mu \qquad \text{......(28)}.$$

The ratio L_m/L_0 is thus an intrinsic property of the material of the ring whatever its linear dimensions.

Equation (26) shows that motional-impedance measurements do not distinguish between an actual moving system and an electrical circuit, consisting of a resistance R_m in series with an inductance L_m and capacity C_m, the whole of which is shunted by the resistance R_c in series with the inductance L_c. As any measurements on the toroidal coil must also include the leakage inductance L_e and the copper resistance R_e the complete electrical circuit revealed by alternating current measurements is that shown in figure 2.

§6. IMPEDANCE DIAGRAM OF MAGNETOSTRICTION OSCILLATOR

A usual means of representing the properties of electromagnetic oscillators is the impedance diagram. The resistance and reactance (or impedance and phase angle) are measured for a series of frequencies and these are plotted in rectangular (or polar) co-ordinates, resistance being measured horizontally and reactance vertically. The points so obtained, when connected by a curve, form the impedance diagram for the oscillator. For a magnetostriction oscillator the curve obtained in the neighbourhood of resonance is practically a circle as shown in figure 6, and if measurements are extended so as to include frequencies well below and then above resonance, the out-of-resonance points lie upon a smooth curve to which the circle is attached. The diameter of the circle which passes through the point of attachment usually slopes downwards at a considerable angle, although this slope may be made small by the use of highly laminated material.

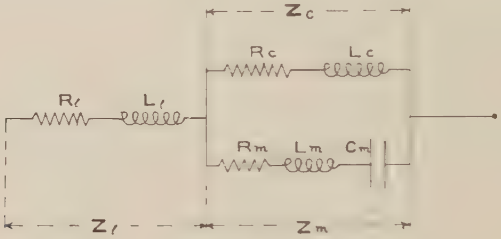

Fig. 2.

X
Y, r
ϕ, l, c, m
T

We will now show that the electrical system of figure 2 will give this type of impedance diagram provided that Z_m is very large, except in the neighbourhood of mechanical resonance. Since resistances are measured horizontally and reactances vertically on the impedance diagram, we will denote resistances by X symbols and reactances by Y symbols, while polar distances will be denoted by r and angles with respect to the horizontal by ϕ. The subscripts l, c and m will as before refer to leakage, core and motion, while the subscript T will refer to the overall components.

The total impedance in vector notation is given by

$$Z_T = Z_l + Z_m Z_c/(Z_m + Z_c)$$

$$= Z_l + Z_c - Z_c^2/(Z_m + Z_c)$$

$$= X_l + jY_l + r_c \epsilon^{j\phi_c} - r_c^2 \epsilon^{2j\phi_c}/(X_c' + jY_c') \qquad \text{......(29),}$$

in which $$X_c' = X_m + X_c, \quad Y_c' = Y_m + Y_c \qquad \text{......(30).}$$

In the immediate neighbourhood of resonance the only factor in (29) which varies considerably with frequency is Y_c', occurring in the last term of (29). Also if the non-resonant value of Z_m is large compared with Z_c, the whole of the effect of this last term occurs in the immediate neighbourhood of resonance. We will assume that the non-resonant value of Z_m is so large that the remaining terms in (29) may be regarded as practically constant during the range of frequency-change in which the last term is appreciable. From (24) and (25) we have

$$Y_c' = \omega (L_m + L_c) - 1/\omega C_m \qquad \text{......(31).}$$

Writing

$$(L_m + L_c)\, C_m = 1/\omega_1^2 \qquad \text{......(32),}$$

and supposing $(\omega - \omega_1)/\omega$ small, we find that

$$Y_c' = 2p\,(L_m + L_c) \qquad \text{......(33),}$$

in which $p = \omega - \omega_1$. p

The pulsatance ω_1 is slightly lower than ω_0, the relation between ω_0 and ω_1 being given by

$$\omega_0/\omega_1 = (1 + L_c/L_m)^{\frac{1}{2}} \doteqdot 1 + \tfrac{1}{2} L_c/L_m \qquad \text{......(34).}$$

In the electrical system shown in figure 2, ω_0 is the resonance pulsatance of the arm Z_m and ω_1 the resonance pulsatance of the mesh $(Z_m + Z_c)$. Mechanically ω_0 is the resonance pulsatance calculated from a value of Young's modulus which is free from the effects of magnetostriction and eddy currents, while ω_1 would be obtained if we included these effects in estimating the modulus. For the purpose of interpretation of the impedance diagram it is preferable to measure variations in pulsatance from ω_1 rather than from ω_0. ω_0 ω_1

If we use the relation

$$\tan \phi_c' = Y_c'/X_c' = 2p\,(L_m + L_c)/X_c' \qquad \text{......(35)}$$

to eliminate Y_c' from (29), we obtain

$$Z_T = X_l + jY_l + r_c \epsilon^{j\phi_c} - \epsilon^{j(2\phi_c - \phi_c')} \cos \phi_c'\, r_c^2/X_c' \qquad \text{......(36),}$$

and ϕ_c' is the only factor which varies with frequency.

On the impedance diagram, figure 3, we first obtain the point M, giving the extremity of the impedance vector at the pulsatance ω_1. Following (36) for the case $\phi_c' = 0$, we draw OL to represent the leakage impedance $X_l + jY_l$ and LC to represent the core impedance $r_c \epsilon^{j\phi_c}$. The last term of (36) is then represented by a line CM of length r_c^2/X_c', $L\hat{C}M$ being equal to ϕ_c. This direction is clearly obtained by rotation of a negative vector through $2\phi_c$ in the positive (or counterclockwise) direction, as is demanded by the form of the phase factor in (36), when $\phi_c' = 0$.

If axes LX', LY' are drawn through L parallel to OX, OY, and CM cuts LX' in P, then $LP = CP$.

When ϕ_c' is not zero the extremity of the impedance vector is shifted to M', which is obtained from M by rotation of CM through ϕ_c' in the negative (or clockwise) direction about C, and reducing its amplitude by the factor $\cos \phi_c'$. Thus the angle $CM'M$ is a right angle and M' traces out a clockwise circle on the diameter CM as ϕ_c' increases.

Through Q, the centre of this circle, draw QS parallel to PL to meet LC in S. Then since $LP = CP$, $QS = QC$ so that the circle cuts LC at S in the vertical direction. The point S is thus the point of minimum resistance of the system.

Again, $CM = r_c^2/X_c' = r_c^2/(X_m + X_c)$, so that CM is always less than r_c^2/X_c or $r_c \sec \phi_c$. But since $CL = r_c$, $CP = \frac{1}{2} r_c \sec \phi_c$, and therefore $CQ < CP$. It follows that the point of minimum resistance always lies above L.

Physically, of course, this result is inevitable, since theoretically X_l may be made as small as we please and the total resistance of the system can never be negative.

The existence of dip of the resonance diameter of the motional-impedance circle is well known, but its inevitable consequence, the limitation of the circle diameter, does not seem to have been sufficiently emphasised. This limitation has an important bearing upon the interpretation of circle diagrams. If the mechanical damping, here represented by X_m, is negligible, then the centre is at P whatever the values of the magnetostriction constants, and this condition is approximated to most closely in the case of solid materials having good mechanical qualities. Mere information as to the sizes of the impedance circles then throws no light upon the magnetostrictive properties of the material. In fact, in the extreme case, the diameter of the impedance circle merely gives a measure of the damped-core impedance. This argument does not apply to measurements upon highly laminated specimens, as in these cases the eddy-current damping, the main factor governing the size of X_c, has been deliberately made small, and the motional factor X_m then has a large measure of control of the circle diameter.

In order to get the value of X_m from an experimental circle diagram it is necessary to know the ratio CP/CQ, which is $(X_m + X_c)/X_c$. This involves a knowledge of the damped-core impedance and therefore of X_c, and the value of X_m follows from CP/CQ.

If the damped-core impedance is not known, we can measure (or calculate) the copper resistance X_l and, knowing CM, we can draw CL bisecting the angle between CM and the horizontal, and make CL cut the vertical through X_l at L. The actual determination of the leakage inductance need be made only in order to verify the adequacy of the theory.

The remaining elements in the arm Z_m may be found if we know the pulsatance change ρ which occurs on passage from M to M', which yields L_m by (35), while the actual pulsatance ω_1 at the point M gives C_m by (32).

§7. DETERMINATION OF MECHANICAL RESONANCE CURVE FROM IMPEDANCE DIAGRAM

I_m If I_m be the current flowing through the arm Z_m of figure 2, then since this is produced by the core voltage v, which $= j\omega N\phi$,

$$I_m = j\omega N\phi/Z_m \qquad \text{......(37)}.$$

Using the value of Z_m given by (23), and the value of ϕ given by (12), we find that I_m is related to the displacement x by the formula

$$I_m = -\kappa x/2N \qquad \text{......(38)}.$$

Thus with the exception of the constant multiplier $2N/\kappa$, I_m gives a measure of x

for all phases of the motion. Now if I be the total current flowing through the system of figure 2, we have

$$I_m/I = Z_c/(Z_c + Z_m) = r_c \epsilon^{j(\phi_c - \phi_c')} \cos \phi_c'/X_c' \qquad \ldots\ldots(39).$$

If, on a current diagram, we draw the vector $r_c \epsilon^{j\phi_c}/X_c'$, this will represent in magnitude and phase the maximum value of I_m/I, and the vector values of I_m/I on either side of this maximum have the locus of their extremities lying on a circle of which the diameter is the maximum vector. We may adapt the impedance diagram of figure 3 to represent the I_m circle by taking the point C as origin and letting the scale be such that CL is numerically equal to I. For in the impedance diagram $CL = r_c$ and $CM = r_c^2/X_c'$, so that CM/CL is equal to the maximum value of I_m/I. Moreover, the vector CM leads the vector CL by the correct phase-angle ϕ_c, so that if the direction of CL indicates the phase of I, then $L\hat{C}M'$ will give the phase-difference between I_m and I. By (38) the circle represents also the displacement diagram for constant total current.

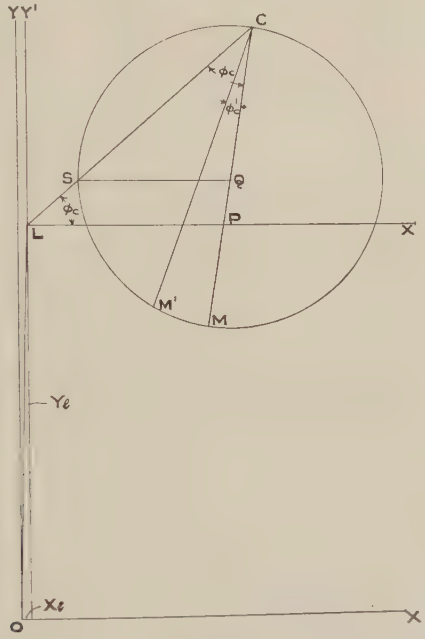

Fig. 3.

§ 8. DETERMINATION OF THE MAGNETOSTRICTION CONSTANTS

By (28) we know that $L_m = L_0 E/\lambda\kappa\mu$ and $L_0/\mu = 2N^2bd/r$, while E is related to the pulsatance ω_0 by the formula

$$\omega_0 = (E/\sigma)^{\frac{1}{2}}/r \qquad \ldots\ldots(40).$$

We have shown how L_m may be found from the impedance diagram, and since the remaining quantities may be determined by simple measurement, we can deduce the value of $\lambda\kappa$. If we accept the relation $\kappa = 4\pi\lambda$ we have thus a means of determining λ. Otherwise we require further information. A determination of the value of x for a known total current would enable κ to be estimated separately by the methods of the last section. The determination of κ, however, in the case of a ring would not be easy, and the experiments of the second part of this paper have been directed to the determination of λ on the assumption that $\kappa = 4\pi\lambda$. The determination of μ and ρ under the working conditions is not too easy for solid rings, but a proper analysis of the change of damped-core impedance with frequency enables these quantities to be estimated. This analysis is described in the second part of the paper and has been carried out in order to see whether the whole theory can be fitted to the facts, while reasonable values of the fundamental constants are still retained.

§ 9. EXPERIMENTAL

The magnitude of the Joule magnetostriction effect in ferromagnetic materials is usually defined by a curve connecting the steady magnetizing field applied to the specimen with the resulting proportional change of length (5, 6). These curves provide some guidance in the choice of a material suitable for a metrical study of the phenomena described in the theoretical work of the preceding section. The changes of induction and the magnetostrictive forces produced by an alternating magnetic field superposed upon a steady magnetic field, which are to be considered here, are not, however, the same as those produced by a small increment of an existing steady magnetizing field, a distinction to be remembered in considering the published curves; the difference between the two cases is somewhat akin to the difference between reversible and differential permeability. The largest magnetostriction effects are observed in nickel (5) and some alloys of the permalloy group (6). These alloys are particularly susceptible to small changes of heat treatment, composition and mechanical working, and cannot be regarded as suitable for a metrical investigation. Nickel has therefore been employed in the following work, since it has been widely studied and many of the properties relevant to this investigation have been measured and can be reproduced by suitable heat treatment.

§ 10. HYSTERESIS AND EDDY CURRENTS IN NICKEL

Part of the theory is concerned with the influence of eddy currents and hysteresis on the magnetostriction effects, and the first step in the experimental work must be to establish the magnitude of these effects for nickel. A circular ring, cut from a sheet of nickel to the dimensions given in figure 1, was annealed at 1000° C. for two hours and cooled slowly during twelve hours. It was covered with a single layer of silk tape and tightly wound toroidally with 231 turns of copper wire of No. 22 s.w.g. insulated to an overall diameter of 0·038 in. The tight winding served a double purpose in that it reduced leakage flux between the wire and the nickel to a minimum and tended to prevent resonance of the ring under the action of the forces produced by magnetostriction. A steady magnetizing field of 14·5 gauss was applied to the ring by direct current flowing in another toroidal winding, after the usual precaution of demagnetizing the ring had been taken. Measurements of the impedance of the tightly wound toroid were made on an alternating-current bridge, described elsewhere(7), over the frequency range 640–41,200 ~. The leakage inductance and the copper resistance were estimated from published formulae(8) and subtracted from the total inductance and resistance in order to obtain the inductance and resistance due to core flux only. This core inductance and resistance can be expressed in the form

$$L_c = L_0 \chi_0 \cos(\eta + \zeta),$$

$$R_c = \omega L_0 \chi_0 \sin(\eta + \zeta) \qquad \text{......(41)}$$

from § 4 (24), and thence

$$R_c/\omega L_c = \tan(\eta + \zeta) \qquad \text{......(42).}$$

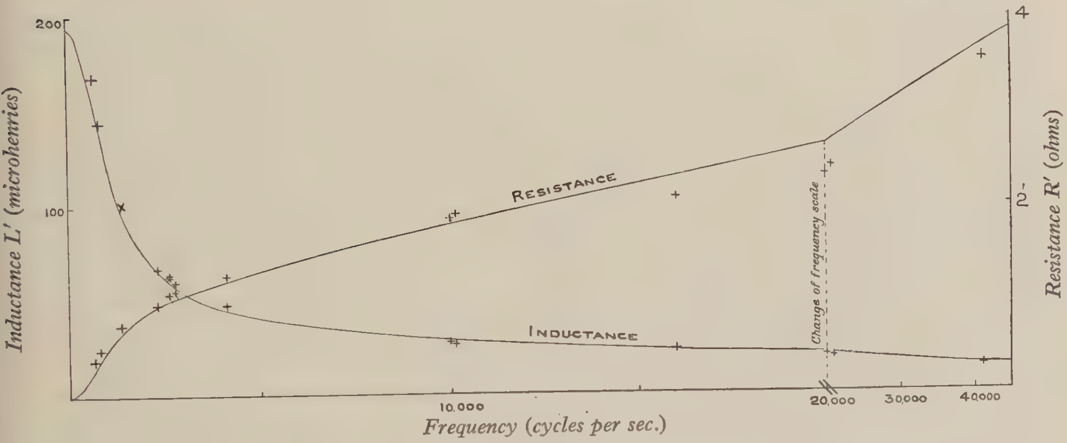

Fig. 4.

Here η and ζ are the angles of lag due to hysteresis and eddy currents respectively. η, ζ The dimensions and properties of the nickel ring are such that flux is confined to a thin surface-layer at frequencies over 5000 ~, and at such frequencies the angle ζ must be 45°. The measured values of $R_c/\omega L_c$ above 5000 ~ averaged 1·15, which is tan 49°, whence $\eta = 4°$.

Now by expanding equation (41), and writing $\sin\eta = \eta$ and $\cos\eta = 1$, we can show that, if small terms in η^2 are discarded,

$$L' = L_0 \chi_0 \cos\zeta = L_c + R_c \eta/\omega, \qquad L'$$

and

$$R' = \omega L_0 \chi_0 \sin\zeta = R_c - \omega L_c \eta \qquad \text{......(43),} \qquad R'$$

where the resistance R' and inductance L' calculated from R_c, L_c and η should be of the same form as the functions $\chi_0 \sin\zeta$ and $\chi_0 \cos\zeta$ respectively. Values of R' and L' are represented by the marked points in figure 4, and values calculated from the functions $\chi_0 \sin\zeta$ and $\chi_0 \cos\zeta$ are represented by the smooth curves. The calculated curves have been brought into the best general agreement by the adoption of the following numerical values for μ and ρ*:

Permeability (reversible)	μ	42·3
Resistivity	ρ	$1{\cdot}00 \times 10^4$ e.m.u.

from which the inductance L_0 at zero frequency is 194 μH and

$$z = 0{\cdot}047\sqrt{f}.$$

It is clear that the theoretical expressions (41) represent the damped impedance of the ring adequately when the symbols have the numerical values given.

§ 11. MOTIONAL IMPEDANCE

The tightly wound toroid and the insulating tape were unwound and the nickel ring was set in a loose-fitting bakelite channel in which it was free to vibrate; the dimensions of this channel are given in figure 1. It was wound with 226 turns of No. 22 s.w.g. copper wire insulated to an overall diameter of 0·038 in. An additional toroidal winding was provided as before to carry a steady magnetizing current adjusted to produce a field of 14·5 gauss. The impedance of the 226-turn winding was measured at a number of frequencies in the range 12,000 to 20,000 ~. The measured impedance is made up of the leakage impedance and the core impedance, and the latter may be regarded as the sum of the damped impedance and the motional impedance†. The leakage impedance is a property of the toroid and has no relation to the magnetostrictive effects; in fact, the core impedance only is required for the verification of the results of the previous section.

The leakage impedance can be estimated from suitable formulae and subtracted from the total damped impedance, but as it amounts to about two-thirds of this impedance the method is not accurate. It is preferable to make use of the results obtained in § 10 and to calculate the part of the damped inductance due to core flux from (41), the appropriate numerical values being used. The two methods give results in satisfactory agreement, thereby providing some check on the accuracy of the measurements. The total impedance due to core flux may now be represented by the smooth curves and the damped impedance due to core flux by the dotted

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* In order to estimate the values of μ and ρ from the observed values of L' and R' the following formulae, deduced from the theory given in § (3), are useful. At high frequencies

$$R' = 2\pi f L' = 2N^2 b\sqrt{(\rho\mu f)}/r \qquad \text{......(A).}$$

At low frequencies

$$\left.\begin{aligned} L' &= L_0\,(1 - \pi^4 t^4 \mu^2 f^2/5\rho^2) \\ R' &= 4\pi^3 t^2 \mu L_0 f^2/3\rho \end{aligned}\right\} \qquad \text{......(B).}$$

From (A) the value of $\rho\mu$ may be found by plotting the high frequency values of R' (or fL') against $\sqrt{f}$. From (B) the value of μ may be found by plotting the low frequency values of L' against f^2. This gives L_0, from which μ may be found. The initial frequency coefficients in (B) can be used as checks.

† The terms "motional impedance," "motional resistance," etc., used in this section must not be confused with the elements of the parallel arm Z_m developed in the theory. They refer to the equivalent series system and are represented in the theory by the last term of equation (36).

curves in figure 5, the difference between corresponding pairs of curves being the motional impedance. The motional inductance or, strictly speaking, the motional reactance increases as the frequency is raised from 12,000 ~, reaching a maximum negative value at about 16,040 ~. It vanishes at 16,490 ~, increases to a small positive value and then falls asymptotically to zero. At 15,970 and 16,110 ~ the motional reactance annuls the damped reactance and the total impedance reduces to a pure resistance; between these frequencies the total reactance is negative, that is, it can be represented by a condenser. The changes of motional resistance can be studied in a similar manner from the curves and do not call for special comment.

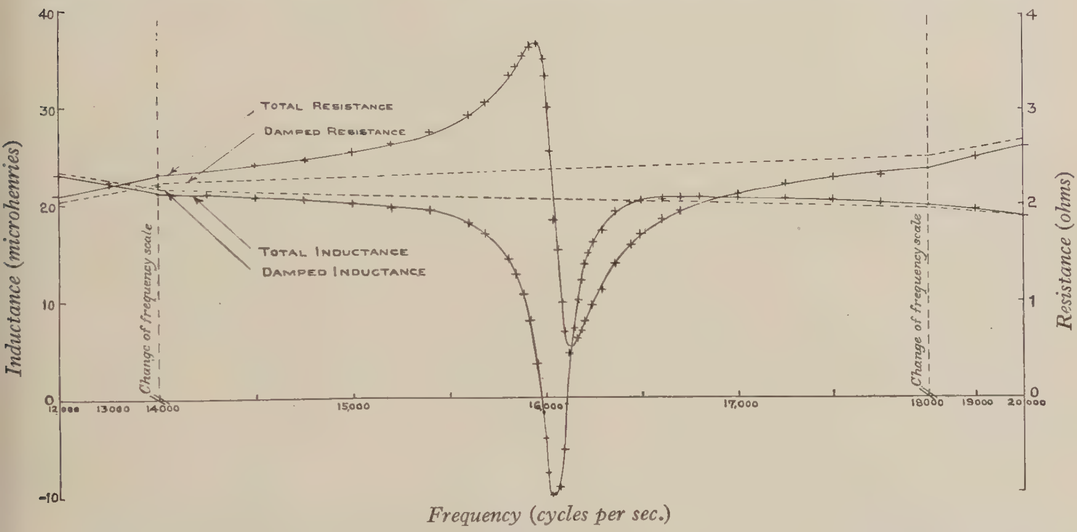

Fig. 5.

§ 12. GEOMETRY OF THE CIRCLE DIAGRAM

The motional impedances corresponding to pairs of marked points in figure 5 can be represented by vectors drawn from the point C in figure 6. The ends of these vectors, which are represented by marked points in the diagram, lie on a circular locus with centre at a point Q and radius QC. The departures of the measured points from the circle are an index of the errors to which the measurements are liable. The resonance diameter CM has been provisionally fixed by the method of plotting, but, as an error in the estimate of damped inductance leads to a movement of C round the circle, it is desirable to devise a more sensitive test for the determination of the exact inclination of CQM to the axes of co-ordinates. If M' is a point on the circle corresponding to a frequency f', equation (35) shows that a straight line should result when tan MCM', that is tan ϕ_c', is plotted against f' for M' and similar points. A diagram of this kind is shown in figure 7 for the correct position of the resonance diameter CM. An error of 0·5° in the inclination of CM destroys the straight-line relation and yields a curve, concave or convex to the axis of frequency according to the sign of the error. The correction to the provisional position of *CM* which results from this test is only $0{\cdot}5^\circ$, which serves as an index of the errors to which the measurement of damped impedance are liable. The final estimate of the angle of depression of *CM* below the horizontal is $98{\cdot}8^\circ$.

A vector *LC* can now be drawn representing the damped impedance at the resonance frequency and, if *L* is taken as origin, vectors drawn from *L* to points on the circle in the neighbourhood of resonance represent the total impedance due to core flux. The restriction is necessary because the length of the vector *LC* is a function of frequency and, strictly speaking, *L* has a different position for every point on the circle. However, a large part of the circle corresponds to a narrow band of frequency, near resonance, for which the position of *L* may be assumed fixed.

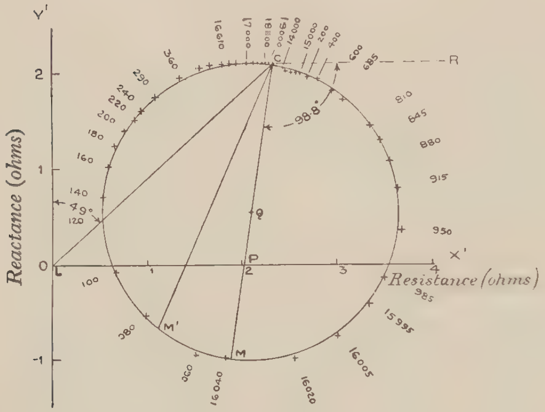

Fig. 6.

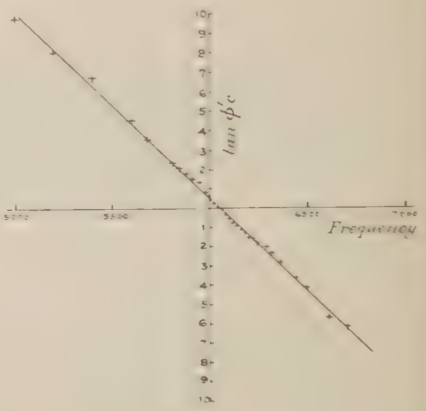

Fig. 7.

The angle CLY' should, according to the theory, be half the angle *RCM* and the triangle *LPC* should be isosceles. The values of the measured angles are as follows: $\angle\, CLY' = 49^\circ$, $\angle\, RCM = 98{\cdot}8^\circ$, $\angle\, CLP = 41^\circ$, $\angle\, LCP = 40{\cdot}2^\circ$, so that the geometrical relations required by the theory are very nearly realized.

§ 13. THE EQUIVALENT ELECTRICAL CIRCUIT

The elements of the equivalent electrical circuit can be derived from figures 6 and 7 with the help of the last two paragraphs of § 6. From figure 6, $X_c = 2{\cdot}37$ ohms, $L_c = 20{\cdot}6\,\mu\mathrm{H}$ and

$$CP/CQ = (X_c + X_m)/X_c = 2{\cdot}12/1{\cdot}57 = 1{\cdot}35,$$

whence $X_m = 0{\cdot}829$ ohms. Now

$$\tan \phi_0'/2p = (L_m + L_c)/(X_m + X_c) = 7{\cdot}75 \times 10^{-4}$$

from figure 7, and thence $L_m = 2{\cdot}459$ mH. Again,

$$\omega_1^2 = 1/(L_m + L_c)\,C_m = (6{\cdot}28 \times 16{,}050)^2 = 1{\cdot}017 \times 10^{10}$$

and

$$C_m = 39{\cdot}6 \times 10^{-6}\ \mathrm{F}.$$

Finally $L_l = 41\,\mu\mathrm{H}$ and $R_l = 0{\cdot}8$ ohms from bridge measurements.

The magnetostriction constants can be derived from the circle diagram or the equivalent electrical circuit. In the latter method the calculation proceeds as follows. From equations (5), (7), and (34)

$$\omega_0^2 = \omega_1^2/(1 + L_c/L_m) = \gamma/\alpha = E/\sigma r^2,$$

which, with a measured value 8·73 of σ, leads to a value for E, the modulus of elasticity, of $2{\cdot}15 \times 10^{12}$. Equations (28) and (3) give the relation

$$L_m/L_\sigma = E/4\pi\lambda^2\mu,$$

whence

$$\lambda = 1{\cdot}76 \times 10^4.$$

§ 14. THE MAGNETOSTRICTION CONSTANT

The circle diagram characteristic of a laminated resonator provides an instructive contrast to that already given for a solid resonator. Nickel rings cut from sheet 0·0025 in. thick, to external and internal diameters of 11·05 and 8·55 cm. respectively, were annealed at 1000° C. for two hours and cooled to room temperature in twelve hours.

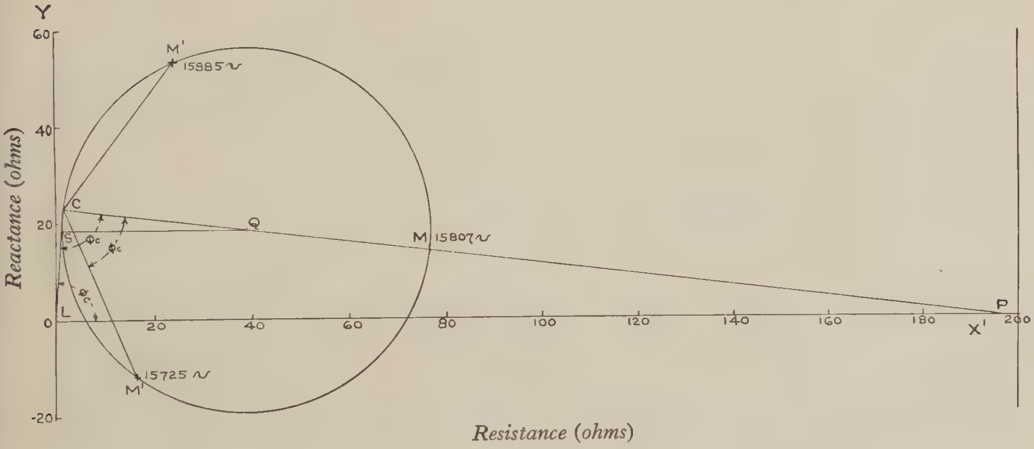

Fig. 8.

The annealed rings were covered on both sides with a thin layer of paraffin wax, piled alternately with paper rings cut from condenser-paper 0·0005 in. thick, and clamped between two thick metal plates. These plates were heated sufficiently to melt the wax and, after slow cooling, the consolidated laminated ring was extracted and freed from superfluous wax. The finished ring was 0·33 cm. thick and weighed 91 gm., corresponding to a density of 7·16 gm./cm³.; it was inferred from the known density of nickel that the composite ring contained 78 per cent. by volume of nickel, the remainder being wax and paper.

The laminated ring was set on three small pieces of spongy rubber at the bottom of a circular-channelled wooden ring. The wooden ring was wound toroidally with 220 turns of No. 24 s.w.g. d.c.c. copper wire insulated to an overall diameter of 0·031 in., the cross-section of this winding, taken to the centre of the wire, being 3·2 cm. in the radial and 1·9 cm. in the axial directions. A second toroidal winding was provided to carry a direct current adjusted to produce a magnetizing field of 14·5 gauss at the mean diameter of the nickel ring.

The impedance of the 220-turn winding was measured at frequencies near resonance on an alternating-current bridge; the technique of this measurement has been described elsewhere by one of the writers(7). The leakage impedance, calculated from the dimensions of the coil, was subtracted from the measured impedances to obtain the core impedance. The latter is represented by the circular locus in the impedance diagram of figure 8, in which the letters correspond to those in figures 3 and 6. The frequencies corresponding to the points M' on the circle such that $CM' = \frac{1}{2}CM$, were found by measurement of a beat note of about 1000 ~ between the valve-oscillator supplying the alternating-current bridge and another valve-oscillator constant in frequency. The effect of eddy currents in the thin sheets can be estimated from the permeability and resistivity of nickel. If $\mu = 38$, $\rho = 10^4$, $f = 15807$ and $d = 0{\cdot}00635$ cm., then $z = 0{\cdot}219$, and from Kennelly's *Tables*(9), $\chi_0 = 0{\cdot}9998$ and $\zeta = 0{\cdot}92^\circ$. The effect of eddy currents is to reduce the integral flux in the ratio 1·0 : 0·9998 and to retard its phase by 0·92°. The angle of depression of the resonance-diameter CM below the horizontal is 6·75° and, since this angle

Table 2.

	1 Symbol	2 Definition	3 Solid ring	4 Laminated ring	5 Published values
Mechanical properties		Inside diameter	8·46 cm.	8·55 cm.	
		Outside diameter	11·06 cm.	11·05 cm.	
	r	Mean radius	4·88 cm.	4·90 cm.	
	d	Axial thickness	0·161 cm.	0·33 cm.	
	b	Radial width	1·30 cm.	1·25 cm.	
	σ	Density	8·73 gm. per c.c.	7·16 gm. per c.c.	8·70–8·91 gm. per c.c.
	f_1	Resonance frequency	16,050	15,807	
	E	Young's modulus	$2{\cdot}15 \times 10^{12}$	$1{\cdot}69 \times 10^{12}$	$2{\cdot}02$–$2{\cdot}35 \times 10^{12}$
Electrical properties	μ	Permeability	42·3	29·7 38 (nickel)	
	ρ	Resistivity	$1{\cdot}00 \times 10^4$ e.m.u.		$1{\cdot}00 \times 10^4$ e.m.u.
	L_0	Inductance at zero frequency	185 μH (for 226 turns)	234 μH (for 220 turns)	
	N	Number of turns	226	220	
Equivalent electrical circuit	R_l	Copper resistance	0·8 ohms	1·43 ohms	
	L_l	Leakage inductance	41 μH	119 μH	
	R_c	Damped core resistance	2·37 ohms	1·36 ohms	
	L_c	Damped core inductance	20·6 μH	234·6 μH	
	R_m		0·829 ohms	5·65 ohms	
	L_m		2·459 mH	12·2 mH	
	C_m		$39{\cdot}6 \times 10^{-9}$ F	$8{\cdot}33 \times 10^{-9}$ F	
Magneto-striction constants	λ	§ 1 (1)	$1{\cdot}76 \times 10^4$	$0{\cdot}934 \times 10^4$	$1{\cdot}77 \times 10^4$* $1{\cdot}53 \times 10^4$†
	H_0	Magnetising field	14·5 gauss	14·5 gauss	14·5 gauss
	B_0	Induction for $H_0 = 14{\cdot}5$	3700	2100	3900*

* See references (5) and (10). † See reference (11).

should be $2(\eta + \zeta)$, it appears that the retardation of phase due to hysteresis is 2·45°.

The elements of the equivalent electrical circuit can be derived from the circle diagram as before, with slight modifications in the calculations to allow for the composite nature of the material. The numerical values for solid and laminated material are summarized in table 2, in which the first column gives the symbol, the second its meaning, or a reference to its first appearance in the text, the third and fourth columns numerical values for solid and laminated material respectively, and the last column numerical values obtained from published tables of physical constants and papers. The numbers above the bar in the fourth column refer to the laminated material and those below the bar have been calculated for nickel from the properties of the laminated material.

Two values of the constant λ are given in the last column, one derived from the work of Masumoto and Nara(5, 10) and the other from the work of McKeehan and Cioffi(11).

§ 15. CONCLUSION

We are now able to review the whole of the experimental work with the help of the diagrams and the preceding table. The values of density, elasticity, and resistivity obtained by measurements on the solid ring show satisfactory agreement with published values. The change of density and elasticity brought about by lamination is clearly indicated in the table, from which it appears that the velocity of sound in the laminated ring is slightly less than that in the solid ring. The magnetostriction constant λ for the solid ring agrees almost exactly with that deduced from the Japanese measurements. The laminated ring gives a smaller value. Some clue to the discrepancy is given by the ordinary B/H curves for the samples, which indicate that the laminated ring is somewhat hardened by mechanical working. The values, given in the table, of induction B_0 when $H_0 = 14{\cdot}5$ gauss show this clearly and justify the view that the small value of B_0 for the laminated specimens may be attributed to this hardness. The authors believe, however, that the close agreement between the value of λ for solid material and that inferred from the Japanese measurements is to some extent fortuitous. The assumption underlying the comparison, that a small change of magnetic induction produces the same magnetostrictive effect independently of the manner in which the change is brought about, can be supported by arguments which are reasonable and plausible rather than strict. It is known that a small alternating field superposed upon a steady field produces an average magnetic induction very slightly different from that appropriate to the steady field acting alone, and similar phenomena are to be expected in the magnetostriction effects, since these are so closely related to magnetic induction. The amount and nature of the impurities present in the samples, the heat treatment and the mechanical working are known to affect the magnetostrictive properties. In fact, precision in magnetostriction measurements can only be attained by use of the purest materials, carefully annealed and free from the effects of cold working. These conditions have been fairly closely fulfilled in the experimental work described here, with the exception of the hardened condition of the laminated specimen, produced by the process of manufacture.

The elements of the equivalent electrical circuit given in the table bring out very clearly the effect of lamination. The corresponding change in the circle diagrams, figures 6 and 8, is also striking. The damping of the solid ring is mainly attributable to eddy currents, that in the laminated ring to mechanical friction, and it is interesting to observe that the solid specimen is not so resonant as the laminated specimen. The circle diagram for the solid specimen, figure 6, approaches the maximum size possible. The limitations imposed by considerations of energy-conservation prevent the excursion of Q along CM beyond the point P, and the maximum radius possible is therefore CP. If we postulate an imaginary material having the same permeability and resistivity but with, say, the magnetostriction constant increased ten times, it would yield a circle but little larger; the size of the circle would not, in fact, reveal this large change in properties. The frequency-spacing round the circle would however change, and the specimen would appear to be much less resonant. Under some conditions, then, sharpness of resonance may be an index of poor magnetostrictive quality. A similar change in the properties of the laminated material would be revealed by a large increase in the size of the circle and only a slight change in the sharpness of resonance, in marked contrast to the solid material. These considerations show that some care is necessary to interpret the circle diagram correctly and that it is essential to discriminate between the parts of the damped impedance due to leakage flux and core flux; that is, the circle diagram should be a diagram of core impedance rather than the usual diagram of total impedance.

The agreement between theory and experiment is uniformly good, such small discrepancies as appear being within the limit of experimental error. The constants for nickel which can be checked against published values are also in good agreement, and it may be concluded that the theory gives an adequate representation of the behaviour of a magnetostriction oscillator in the form of a ring.

§ 16. ACKNOWLEDGMENT

The authors desire to express their indebtedness to the Admiralty for permission to publish this paper.

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DISCUSSION

Dr. D. W. Dye. I wish to congratulate the author upon the elegant manner in which he has developed the equivalent electric circuit of the magnetostriction oscillator. These equivalent circuits are of the greatest value to those who have to work in terms of associated mechanical vibrating systems and electrical circuits. Experiments which I communicated to the Society a few years ago on piezo-electric resonators proved that the equivalent circuit for this case can be applied with very great accuracy.

The ring form of oscillator, such as that described in the paper, is of a particularly satisfactory kind since, in an isotropic material, the stresses and strains are uniform throughout the ring from section to section. For this reason I have been experimenting with rings cut from natural quartz. These cut with plane perpendicular to optic axis have uniform properties in any azimuth in the ring, and when supplied with suitably disposed electrodes can be made to vibrate radially in the fundamental mode. It was found however that the damping was surprisingly large. On examination of the numerical values given in the paper in table 2 it will be found that for the magnetostriction resonators the logarithmic decrement given by the ratio $Rm/2L_m n$ has the value 0·01 for the solid ring, and about the same value for the laminated ring.

This is about the same as for a quite moderately small electrical circuit. I have found that in the case of the quartz ring the damping was nearly all due to the air-viscosity between the cylindrical surfaces and the closely embracing electrodes. When the ring was placed in a vacuum this damping all disappeared and the ring behaved as a first-class resonator. It occurs to me that a large part of the heavy damping of the magnetostriction ring might be due to the same cause since the ring is pocketed in an enclosure closely embracing it.

THE THEORY OF THE MICROSCOPE

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ABSTRACT. The paper examines the diffraction-effects produced by (*a*) two adjacent apertures, and (*b*) a series of apertures in an opaque screen situated in the focal plane of a lens system, when the illuminating system is projecting the elementary image of a point-source of light into this object plane. The diffraction-effects and geometrical resolving power of the grating are shown to be independent of the concentration of the light in the object plane; they depend rather on the number of apertures free to transmit light. The theory is then extended to the case where the illumination of the object is produced by a source of finite area. Both the equivalence and Abbe principles appear in the analysis of the mode of formation of the image in such critical illumination, but subject to the difficulties encountered in integrating for the effects of finite sources of light. Confirmatory experimental work is in hand.

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§ 1. INTRODUCTION

The recent renewal of interest in photomicrography by ultra-violet radiation has re-directed attention to the problem of the interpretation of the microscope image, and hence to the question of the mode of formation of the image. The work of E. Abbe, and the theory associated with his name, sprang from the realization that the physical process involved in the formation of the image of a non-self-luminous object must (in the microscopic sense at least) differ radically from the action in the case of a self-luminous body. Practical experience leads, however, to the conclusion that in the final result there may be a great measure of equivalence between the two cases. Thus the late Lord Rayleigh investigated* the image of a row of points and of a grating, in cases both where they are assumed to emit light of equal intensity in the same phase, and where they are self-luminous. The case where the phases of the successive elements have a constant difference is also discussed. In two or three paragraphs he discusses the function of the condenser when it projects an image of the source of light into the object plane: "...To a point of the flame corresponds in the image, not a point, but a disk of finite magnitude....The radius of the disk has the value $\frac{1}{2}\lambda$ sin α, where α is the semi-angular aperture of the condenser....It seems fair to conclude that the function of the condenser in microscopic practice is to cause the object to behave, at any rate in some degree, as if it were self-luminous, and thus to obviate the sharply-marked interference-bands which arise when permanent and definite phase-relations are permitted to exist between the radiation which issues from various points of the object." This opinion, which was not the result of any detailed investigation but was clearly expressed as an opinion, is the accepted basis of the use by practical microscopists of the so-called "critical illumination" in which the image of the radiant is formed in the plane of the object; it is further considered by many that the object behaves under such conditions as if there were then practically no phase-relations amongst the light emitted from the various parts and that *all* interference effects are thereby avoided.

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* *Phil. Mag.* **42**, 167 (1896).

A further conclusion is sometimes drawn that the Abbe principle is valueless in the practical circumstances of observation in the microscope. For these reasons it seems that perhaps a closer discussion of this case of "critical illumination" is desirable.

§ 2. THE CASE OF TWO APERTURES

The case of two indefinitely small apertures may be recalled for the purpose of illustrating the present discussion. Let the dispositions be indicated in figure 1, where S is the object plane and C is the condenser supposed free from aberration.

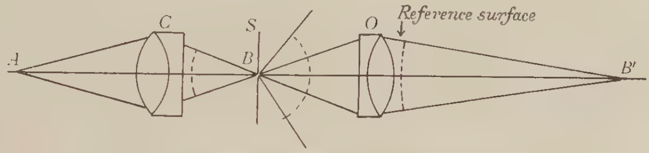

Fig. 1. Critical illumination (diagrammatic).

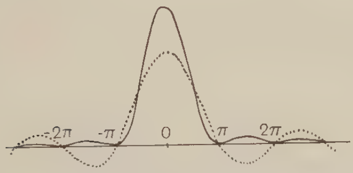

Fig. 2. Relative amplitude ($\sin u/u$) (dotted) and intensity ($\sin^2 u/u^2$) (full line.)

Consider first an elementary axial source of light at A. The image projected into the object plane is not merely a disc; it consists of the well-known distribution of light, first worked out by Airy, in which the amplitude variation along a radial direction is represented by

$$J_1(d)/d,$$

where d is proportional to the radius. The relative intensity and amplitude are very similar to those plotted in figure 2; the amplitude in the first bright ring surrounding the central disc is as much as one-eighth of the central maximum; although the phases of the odd number rings differ by π from that of the central disc and even rings, yet the phase is otherwise very nearly constant* over the various regions. This is equivalent to the assumption of negative amplitudes for the odd rings†.

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* Rayleigh, *loc. cit.* p. 1; L. C. Martin, *An Introduction to Applied Optics*, p. 95 (Pitman and Sons, London, 1930).

† This attribute of the Airy disc is so generally misunderstood that a simple discussion of the point has been added to the paper as an appendix.

Given two elementary apertures M and N situated symmetrically near B, figure 1 (see also figure 5), and assuming that the illuminating disc falls on them symmetrically, we see that they will radiate in the same phase and with equal intensity, while the radiation into various angular directions will be fairly uniform if the holes are small enough. If the objective O is free from aberration, each aperture by itself produces an image of the Airy disc type in the plane through B'. If two co-phasal images overlap, the displacements (and thus in our case the amplitudes) have to be added in order that we may realize the resultant effect; if the light in the two apertures M and N had been non-coherent the intensities would have had to be added directly.

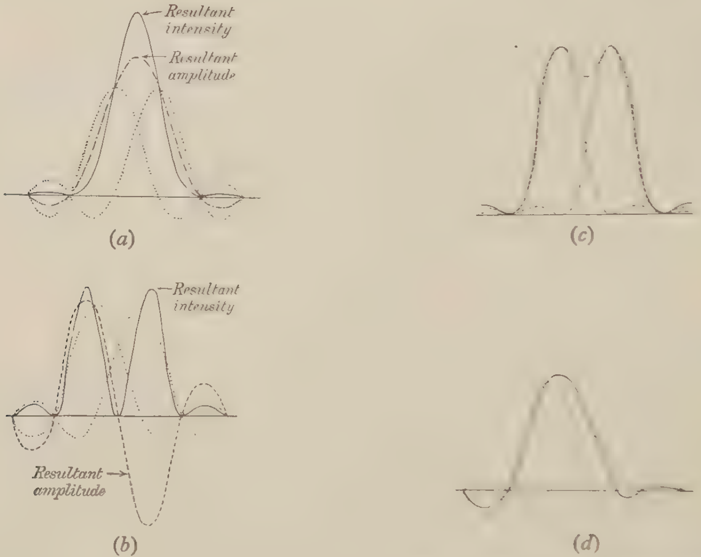

Fig. 3. Superposition of two elementary images. (*a*) Light in same phase; amplitudes are added. (*b*) Light in opposite phase; amplitudes subtracted. (*c*) Light not coherent; intensities are added. (*d*) Light in same phase; amplitudes unequal.

Let the case be chosen when the separation of the centres of the images is equal to the radius of the first dark ring of the Airy disc. The result (co-phasal case) is illustrated in figure 3*a* for the intensity-distribution along the line through the two centres. If the original point-source is moved to a place such that M and N are in opposite phase, the amplitudes are to be subtracted, giving the amplitudes of figure 3*b**.

It will easily be seen that there is no separation of maxima in the resultant if the apertures emit light in the same phase. A little graphical work allows the contour diagram of intensity in the plane of B' and for the area immediately surrounding B' (due to the light from the two apertures M and N when they are symmetrically illuminated) to be drawn, figure 4. The patch of light is indeed elongated in one direction but shows no sign of its double nature as it does in the case when the light in the images is incoherent, figure 3*c*. On the other hand there is a marked division between two maxima in the resultant if the apertures emit light in opposite phases. Figure 3*d* suggests the case of light in the same phase but unequal intensity.

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* The amplitude-distribution curve in the figures actually relates to the distribution characteristic of a rectangular aperture; this fact produces no important alterations in the general results. Figure 4 is computed for the true circular aperture distribution.

With these results in mind we may illustrate the application of something like the Abbe principle for this case. From the above condition of separation of the image points it follows that the separation h of the corresponding object points is given by h

$$h = 0{\cdot}61\lambda/\mathcal{N}\mathcal{A}_0,$$ λ,

where $\mathcal{N}\mathcal{A}_0$ is the numerical aperture of the objective. The distribution of light in the posterior principal focal surface of the objective (constructed for very close parallel rays derived from the neighbourhood of B) now resembles that of a system $\mathcal{N}\mathcal{A}_0$

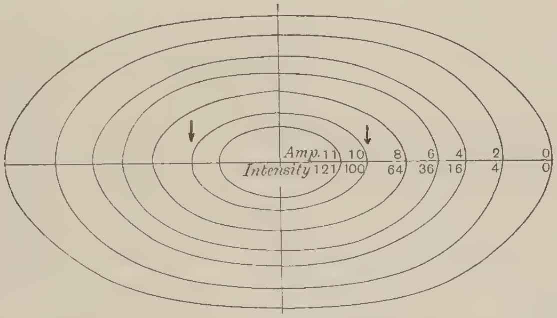

Fig. 4. Contour map of intensity distribution in the final image of adjacent small apertures illuminated by coherent light. Separation of apertures $=0{\cdot}61\lambda/\mathcal{N}\mathcal{A}$. Positions of individual maxima indicated by arrows. (Circular apertures.)

of simple interferences. Let α be the angle made by such rays with the axis in the axial plane containing the apertures; then the resultant amplitude A in the corresponding point F of the focal surface, figure 5, is given by α

$$A = 2a_1 \cos\{(\pi nh \sin\alpha)/\lambda\} \qquad \text{......(1)},$$

where a_1 is the amplitude due to one hole and n is the refractive index of the object space. Naturally the amplitudes of adjacent bright fringes alternate in sign. a_1, n

We may now go a step further. The mutual angular divergence of the rays passing through F is so small that the relative path-differences of the disturbances meeting in the corresponding points of a spherical reference surface centred at B' will not be appreciably different from those at F. Since the lens would form at B' the image of an elementary source at B, the phases of the disturbances from such a source at B would be identical all over the reference surface. Also, owing to the symmetry, the phase of the resultant disturbance due to the two apertures M and N will be identical with that due to an imaginary source at B, the mid-point; hence the phase of the resultant will be uniform over the reference surface.

A proportion of this interference system, depending on the aperture as suggested
u in figure $6a$, $6b$ is focussed behind the objective. When u is the extreme
divergence for a ray entering the objective $n \sin u =$ [illegible], so that, if we take the value of h given above, we obtain

$$\text{(Marginal) } A = 2a_1 \cos (0{\cdot}61\pi),$$

in which case a small part of the second bright fringe on each side of the central one begins to be included. The integration below is carried out for a wider aperture.

The calculation of the resultant energy distribution in the image plane on the basis of Huygens's principle can be carried out without difficulty for a rectangular aperture with edges parallel and perpendicular to the fringes. In order to obviate some difficulties the aperture parallel to the directions of the fringes is considered small. In a familiar manner, the effect of the whole aperture is reduced to that of a strip in the plane of the aperture and parallel to the diametral direction along which the integration is to be effected i.e. perpendicular to the fringes. Such a strip is shown by PAQ, figure 7, in which A is the centre of the strip and B' is the centre of curvature. The points R and S show equal and equidistant elements
dy dy, and the point C in the focal surface is conveniently defined by the difference of
ϕ phase 2ϕ with which disturbances from P and Q meet in it. Again the resultant
phase for all pairs of elements giving equal contributions, such as those at R and S, will be sensibly constant, since there is equal lag and lead for the phases of the disturbances from elements symmetrically situated*.

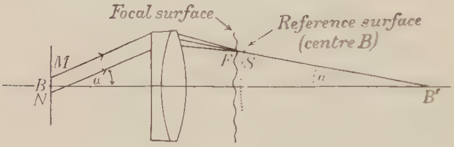

Fig. 5.

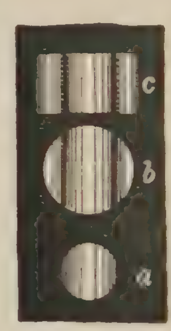

Fig. 6.

Fig. 5. Focal surface and reference surface.

Fig. 6. To illustrate diffraction or interference maxima found in reference surface of objective when two apertures on stage are illuminated by coherent light.

y, Y Let the distances AR, AP be y, Y; the effective amplitude of the displacement
A_y at R and S be A_y. The phase-difference of the disturbances arriving at C from these
δ points is δ, where

$$\delta = 2\phi y/Y.$$

Hence the resultant amplitude at C due to the elements at R and S is

$$\text{constant} \times 2A_y dy \cos (\phi y/Y).$$

---

* See Appendix.

But A_y must be calculable from the above expression in equation (1). Since the optical sine relation gives

$$nh \sin \alpha = n'h' \sin \alpha',$$

n', h', α'

then

$$A_y = 2a_1 \cos \{(\pi n'h' \sin \alpha')/\lambda\}^*,$$

thus for the purposes of our problem, since $\sin \alpha'$ is proportional to y, we can replace $(\pi n'h' \sin \alpha')/\lambda$ by $\Delta . y/Y$ and write

$$A_y = 2a_1 \cos (\Delta . y/Y),$$

where Δ is half the phase difference, at the ordinate Y, of the disturbances from M and N, figures 1 and 5, and the final expression for the amplitude at C is

Δ

$$\text{const.} \times \int_{y=0}^{y=Y} \cos (\Delta . y/Y) \cos (\phi y/Y)\, dy.$$

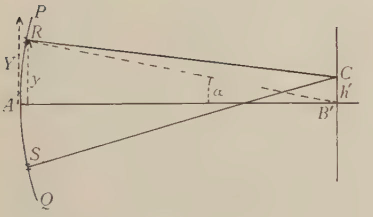

Fig. 7.

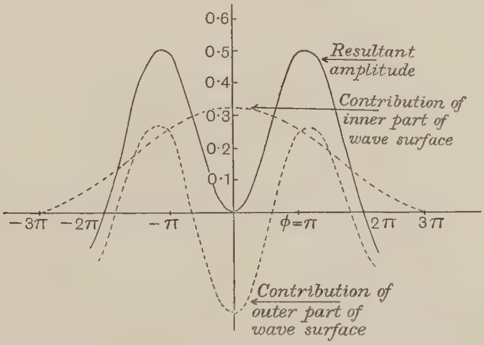

Fig. 8.

Fig. 8. Showing the amplitude-contributions of the inner and outer parts of the wave surface for points near the axis when the object consists of two small apertures symmetrically illuminated by co-phasal light.

The case now chosen for integration is that in which the aperture extends to the first lateral maximum on each side in the reference surface, i.e. $\Delta = \pi$, so that h becomes equal to $\lambda/\mathscr{N}\mathscr{A}_0$. The integral can then be written

$$\text{Amplitude} = \frac{1}{2}\int_0^Y [\cos \{y(\pi - \phi)/Y\} + \cos \{y(\pi + \phi)/Y\}]\, dy$$

$$= \frac{Y}{2}\left[\frac{\sin \{y(\pi - \phi)/Y\}}{\pi - \phi} + \frac{\sin \{y(\pi + \phi)/Y\}}{\pi + \phi}\right]_0^Y$$

$$= \frac{Y}{2}\left\{\frac{\sin (\pi - \phi)}{\pi - \phi} + \frac{\sin (\pi + \phi)}{\pi + \phi}\right\} \quad \text{......(2)},$$

for which the values are easily computed for a series of values of ϕ. They are plotted in figure 8 in full lines.

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* See Appendix.

The effect of the central and marginal parts of the aperture can be illustrated by separation of the integral into the two parts

$$\int_0^Y = \int_0^{Y/2} + \int_{Y/2}^Y,$$

with the following result

$$\frac{Y}{2}\left\{\frac{\sin\frac{1}{2}(\pi-\phi)}{\pi-\phi}+\frac{\sin\frac{1}{2}(\pi+\phi)}{\pi+\phi}\right\}$$
$$+\,Y\left\{\frac{\cos\frac{3}{4}(\pi-\phi).\sin\frac{1}{4}(\pi-\phi)}{\pi-\phi}+\frac{\cos\frac{3}{4}(\pi+\phi).\sin\frac{1}{4}(\pi+\phi)}{\pi+\phi}\right\}.$$

The two parts are represented for various values of ϕ by the dotted lines in figure 8, from which the inner part of the wave-surface is seen to produce no resolution of the images. The co-phasal regions at the margins produce an interference-system of which the maxima are spaced by a separation rather greater than the separation of the maximum and minimum in an elementary image resulting from the action of the whole aperture. Also the side fringes in the focal plane of O will have a phase difference of π from the central fringe*. Hence at B' the contribution of the outer part of the wave-surface is in opposite phase to that due to the central region. Thus it is that the interference system passes for the resolved "images" of the two elementary apertures.

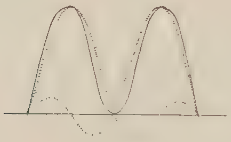

Fig. 9.

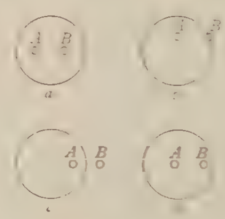

Fig. 10.

Fig. 9. Superposition of two elementary images separated by $\phi = 2\pi$.

Fig. 10. Showing how the illuminating Airy disc (first dark ring indicated by the large circle) may fall on two small adjacent apertures A and B.

We reach precisely the same result if the amplitude of two co-phasal elementary images, having their maxima separated by an interval ϕ of 2π, are simply added together. The amplitude in such a single image (due to a rectangular aperture) is represented by the general function $\phi^{-1}\sin\phi$. The addition for the separation 2π is shown in figure 9. It is also clear from equation (2).

The above work illustrates one of the main conceptions of the Abbe theory, namely the production of an interference phenomenon in the posterior focal plane of the objective from which the nature of the final image may be deduced. In the above simple case, however, the actual areas characterized by uniform phase are finite, and not merely sets of corresponding lines or points as are the "homologous points" usually discussed in the Abbe theory.

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* See Appendix.

§3. ILLUMINANT OF FINITE AREA

It is seldom or never the case in practice that the source of illumination approximates to a point. In "critical illumination" the source consists of an assembly of the point-sources, as considered above, of which the elementary images will be assumed to be in focus in the plane of the "object": let the latter again consist of two elementary apertures. Let the separation of these be so close that they are on the limit of resolution, and let the aperture of the condenser be increased to equal that of the objective. The radius of the Airy disc corresponding to each elementary point of the source will then be approximately equal to the separation of the apertures, which will be illuminated by discs falling over them in all possible ways. Some cases are shown in figure 10, which suggests various ways in which the Airy disc may fall upon the two apertures A, B; the line represents the dark minimum (circular aperture). These cases are as follows:

(*a*) Both apertures illuminated by light of equal intensity and in the same phase, figure 10*a*.

(*b*) One aperture only effective, the other being situated on the dark circle, figure 10*b*.

(*c*) Equal intensity; phases opposite*, figure 10*c*.

(*d*) Unequal intensity; phases agree, figure 10*d*.

We should expect that, when the images are on the point of resolution, the central maximum of one image would fall on the first dark ring of the other if the object points were self-luminous. A number of cases corresponding to the conditions above are shown in figure 3. Figure 3*a* corresponds to condition (*a*). The phases agree, the intensities are equal, and there is no division of the maxima. Figure 3*b* corresponds to case (*c*), in which the phases are opposite. The amplitudes of the elementary images are now subtracted, and it is evident that there is a sharply marked separation of the maxima. Figure 3*d* shows a case of unequal intensity with agreement of phase.

The final result will be the statistical average of all possible cases, and without closer discussion it will be seen that the ultimate effect is likely to be similar to that characteristic of two independent elementary sources, the resolution being made possible by the fact that some at least of the light from A and B emerges in opposite phases. The sum of the intensities for two elementary images is shown in figure 3*c*.

It will be understood from the above that, while the action can be discussed much more simply from the point of view of the equivalence principle, the emergence of this principle has not involved any new physical conceptions. The more detailed analysis, first contemplating the effects in the principal focal surface of the objective, then proceeding to the effects of the final image, still remains valid.

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* With regard to (*c*), note that the phase of the first bright ring in the elementary image is opposite to that of the first dark minimum and the point B is supposed to lie on the first bright ring while A is still on the central disc.

§4. CASE OF THE GRATING; POINT-SOURCE OF ILLUMINATION

The next problem concerns the case of the grating, represented in an ideal scheme by a row of elementary apertures. This object lies on the stage of the microscope and we will assume that the illumination is furnished by a condenser of rectangular aperture with its principal direction parallel to the line of elements. The source of illumination first considered is an axial elementary point-source of which the image falls into the object plane, giving an amplitude-distribution of the type plotted in figure 2, and represented by an equation of the general form

$$A = \frac{\sin \phi}{\phi}.$$

We shall consider later how the results are affected by the use of an extended source of illumination. Note that the first lateral minimum occurs where the difference of phase 2ϕ between the disturbances arriving in the illumination image from the extremities of the condenser aperture is equal to 2π, and the linear distance of this minimum from the central maximum will be given by

$$h = 0{\cdot}5\lambda / \mathcal{N}\mathcal{A}_c,$$

$\mathcal{N}\mathcal{A}_c$ where $\mathcal{N}\mathcal{A}_c$ is the numerical aperture of the condenser.

As a first simple case, assume the object B to consist of a row of apertures situated in the object plane symmetrically about the central maximum. The relative amplitudes of the light issuing from successive apertures in both directions must therefore be given by such terms as

$$\frac{\sin u}{u}, \quad \frac{\sin 3u}{3u}, \quad \frac{\sin 5u}{5u}, \text{ etc.},$$

where u depends on the spacing. The final resultant of the light diffracted in the plane containing the apertures and the axis into any given direction will be found from

$$A^2 = [\Sigma \{a \sin \delta + a \sin (-\delta)\}]^2 + [\Sigma \{a \cos \delta + a \cos (-\delta)\}]^2,$$

since there will be equal lag and lead in the phases of elements on each side of the centre. Hence as usual,

$$A = \Sigma 2a \cos \delta.$$

x Let x be the grating interval, then the first pair contribute an amplitude (into the direction making an angle θ with the axis) given by

$$2a_1 \cos \delta = 2\left(\frac{\sin u}{u}\right) \cos\left(\frac{\pi}{\lambda} x \sin \theta\right),$$

and the second pair give

$$2 \frac{\sin 3u}{3u} \cos\left(\frac{\pi}{\lambda} 3x \sin \theta\right).$$

The whole effect of the grating is that

$$\text{Amplitude} = 2\left\{\frac{\sin u}{u} \cos\left(\frac{\pi}{\lambda} x \sin \theta\right) + \frac{\sin 3u}{3u} \cos\left(3\frac{\pi}{\lambda} x \sin \theta\right) + \frac{\sin 5u}{5u} \cos\left(5\frac{\pi}{\lambda} x \sin \theta\right) + \text{etc.}\right\},$$

the number of terms included depending on the number of open apertures, or if v be written for $(\pi/\lambda)\, x \sin\theta$,

$$\text{Amplitude} = 2u^{-1}\left(\sin u \cos v + \tfrac{1}{3}\sin 3u \cos 3v + \tfrac{1}{5}\sin 5u \cos 5v + \text{etc.}\right)$$

$$= u^{-1}\left[\{\sin(u+v) + \tfrac{1}{3}\sin(3u+v) + \text{etc.}\} + \{\sin(u-v) + \tfrac{1}{3}\sin(3u-v) + \text{etc.}\}\right] \quad \ldots\ldots(3).$$

These series represent well-known Fourier expansions. The value of the first bracket, if an infinite number of terms are included, is $\pi/4$ from $(u+v) = 0$ to $(u+v) = \pi$, and then $(-\pi/4)$ from $(u+v) = \pi$ to $(u+v) = 2\pi$ and so on. The second bracket, depending on $(u-v)$, is similarly evaluated. In order to understand the result we may select special cases. Take for example $u = \pi/2$, which means

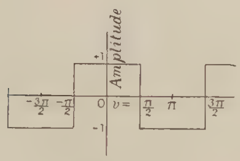

Fig. 11.

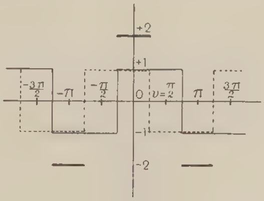

Fig. 12.

Fig. 11. Distribution of amplitude in the back focal surface of the objective when $u = \pi/2$.

Fig. 12. Distribution of amplitude when $u = \pi/4$.

that the value of u corresponding to the first lateral aperture is equal to $\pi/2$, so that this aperture falls well within the illuminating disc. The effect is most quickly seen by substitution in the single form of the series above, whence we obtain

$$\text{Amplitude} = 4\pi^{-1}\left(\cos v - \frac{\cos 3v}{3} + \frac{\cos 5v}{5} - \text{etc.}\right),$$

i.e. the cosine development of unity, of which the value is $(+1)$ from $v = -\pi/2$ to $+\pi/2$, and (-1) on each side, figure 11. We have, in fact, the case where the distribution of light in the focal surface of the objective shows the broad central maximum representing the direct image of the entrance pupil of the condenser system, sharply bordered by a succession of lateral diffraction maxima having amplitudes of alternating sign.

If the aperture of the condenser is reduced, the images will no longer touch each other. The way in which this arises is illustrated in figure 12, exhibiting the case when $u = \pi/4$, corresponding to the case of half the previous condenser aperture. The value of the first series in equation (3) is $\pi/4$ from $v = (-\pi/4)$ to $v = 3\pi/4$, and is $(-\pi/4)$ from $v = 3\pi/4$ to $v = 7\pi/4$ and so on; the value of the second series is $\pi/4$ from $v = (-3\pi/4)$ to $v = \pi/4$ and so on. Adding the amplitudes, we arrive at regions of amplitude 2, alternatively positive and negative, separated by regions of no disturbance. These regions correspond to the separated images of the condenser stop found in this case.

On the other hand it is interesting to study the effects of overlapping of the diffraction images. When $u = 3\pi/4$, the first series represents $\pi/4$ from $v = (-3\pi/4)$ to $v = \pi/4$, and is periodic in the usual way; the second series is $\pi/4$ from $v = 3\pi/4$ to $(-\pi/4)$ and so on. The addition of these results seems at first sight to contradict the expected results; it shows separated maxima as in figure 12 and does not seem to indicate any overlapping, such as is frequently observed when diffraction images are actually made to overlap in the microscope by opening of the stop of the condenser. We have only, however, to consider what will happen if the overlapping images (width $3\pi/2$) are of equal intensity and opposite phase to realize that the result of the analysis is correct. The usual appearance in the microscope arises because there is not the complete correspondence of phase characteristic of this ideal case, nor the exactly equal intensities now imagined between the direct and diffracted beams.

The above cases contemplate a position of the grating which has the centre of a dark space on the axis. If a bright aperture is situated on the axis the series is no longer the same. We then obtain a series of the form

u_1, v_2

$$A = 1 + u_1^{-1} (\sin u_1 \cos v_2 + \tfrac{1}{2} \sin 2u_1 \cos 2v_2 + \tfrac{1}{3} \sin 3u_1 \cos 3v_2 + \text{etc.}),$$

where v_2 is now $(2\pi/\lambda)\, x \sin\theta$.

Expanding as before we can separate two series from the odd coefficient terms and two from the even:

$$A = 1 + 2u^{-1}\{(\sin \overline{u_1 + v_2} + \tfrac{1}{3} \sin \overline{3u_1 + v_2} + \text{etc.}) + (\sin \overline{u_1 - v_2} + \tfrac{1}{3} \sin \overline{3u_1 - v_2} + \text{etc.})$$

$$+ (\tfrac{1}{2} \sin \overline{2u_1 + v_2} + \tfrac{1}{4} \sin \overline{4u_1 + v_2} + \text{etc.}) + (\sin \overline{2u_1 - v_2} + \tfrac{1}{4} \sin \overline{4u_1 - v_2} + \text{etc.})\}.$$

The first two series are the familiar Fourier expansions involving finite discontinuities of the type dealt with above. The second two series are not so easily discussed mathematically, but involve sudden discontinuities of sign at values 0, π, 2π, etc., of the parameter. In view of the general theory to follow they need not be discussed, but when $u = \pi/4, \pi/2, 3\pi/4$ etc., they reduce to specially simple forms, which will illustrate the case. Clearly when $u_1 = \pi/2$, or any integral multiple of $\pi/2$, the series of even terms will cancel. Again when $u_1 = \pi/4$ the even terms of the series in its first form are

$$= \tfrac{1}{2} \cos 2v_2 - \tfrac{1}{6} \cos 6v_2 + \tfrac{1}{10} \cos 10v_2 + \text{etc.}$$

$$= \tfrac{1}{2} \{\cos 2v_2 - \tfrac{1}{3} \cos 3\,(2v_2) + \tfrac{1}{5} \cos 5\,(2v_2) + \text{etc.}\},$$

a Fourier expansion again involving sudden discontinuities. The same series with reversed sign appears when $u_1 = 3\pi/4$. The various particular cases are very interesting to work out in detail. When $u_1 = \pi/4$, the presence of the constant positive term of value unity suppresses the maxima of negative amplitude, and the positive ones only are left. Absent spectra may thus occur, even in the presence of critical illumination, and may furnish the explanation of an experiment shown to the writer by Mr Conrad Beck, in which the resolution of a grating entirely disappeared when the illumination was exactly critical, reappearing for other adjustments of the condenser. The particular cases when u_1 is any integral multiple of π are the only cases in which the central aperture alone is illuminated and the amplitude is uniform in the back focal surface. This is the complete realization of the equivalence principle, but in a very particular case.

§ 5. ASYMMETRICAL ILLUMINATION OF THE GRATING

In general, the apertures of the grating will be asymmetrically illuminated by the condenser, even though a single elementary illuminating element is considered. If the maximum illumination lies asymmetrically with regard to any two elements we may suppose that the apertures lie at the points given by

$$u, \quad u \pm v, \quad u \pm 2v, \quad \text{etc.},$$

where v is now a measure of the grating interval. The series representing the v
resultant amplitude must now take account of the phase differences of the disturbances from the successive apertures, which is directly proportioned to the sine of the angle of obliquity and to the spacing. Hence the series can be written

$$\text{Amplitude} = \frac{\sin u}{u} + \frac{\sin(u+v)}{u+v}e^{-imv} + \frac{\sin(u-v)}{u-v}e^{+imv} + \frac{\sin(u+2v)}{u+2v}e^{-2imv} + \text{etc.},$$

where m depends on the angular inclination; see below. It is of interest to note m
that exactly the same series was employed by Lord Rayleigh* to represent the resultant amplitudes in the microscope image when the object is a grating illuminated by plane waves incident at various angles, although the terms then bear another significance. In that paper Rayleigh points out that if the series be multiplied by the factor e^{-imu} it becomes periodic with respect to u in period v, and hence can be developed by Fourier's theorem in periodic terms, in the form

$$A_0 + iB_0 + (A_1 + iB_1)\cos(2\pi u/v) + (C_1 + iD_1)\sin(2\pi u/v) + \dots$$
$$+ (A_r + iB_r)\cos(2\pi ru/v) + (C_r + iD_r)\sin(2\pi ru/v) + \dots.$$

The multiplication of the terms of the series by e^{-imu} (interpreted in our case) simply refers the phase of the resultant to that of an imaginary disturbance propagated from the centre of the illuminating disc, which may not, however, be exposed. The series is then

$$\frac{\sin u}{u}e^{-imu} + \frac{\sin(u+v)}{u+v}e^{-im(u+v)} + \frac{\sin(u-v)}{u-v}e^{-im(u-v)}$$
$$+ \frac{\sin(u+2v)}{u+2v}e^{-im(u+2v)} + \text{etc.}$$

The full discussion cannot be reproduced here, but the resulting expansion appears as a series of terms which exist only between certain values of the variables. Writing

$$s_r = 2\pi r/v,$$

we find that B_r and C_r both become zero, so that the series, being left as a succession of terms involving ($\cos \pm i \sin$), can be expressed exponentially.

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* *Loc. cit.* p. 1.

The rth term of the above Fourier expansion appears as

$$\frac{\pi}{2v}\left[e^{is_r u}\{(1+m+s_r)^* + (1-m-s_r)^*\} + e^{-is_r u}\{(1+m-s_r)^* + (1-m+s_r)^*\}\right],$$

* in which the meaning of the asterisk against each bracket is that this bracket is subject to a certain limitation; it only appears as (− 1), the sign being that of the bracket. Hence the term in $e^{is_r u}$ in the series vanishes unless $(m + s_r)$ lies between (+ 1) and (− 1), and the term in $e^{-is_r u}$ vanishes unless $(m - s_r)$ lies within the same limits. The first term has

$$B_0 = 0,$$
$$A_0 = (\pi/2v)\{(1+m)^* + (1-m)^*\},$$

A_0 vanishing unless m lies between (+ 1) and (− 1), as above. Then when A_0 exists, its value is π/v, so that the whole series is represented by

$$(\pi/v)\,(1^* + e^{is_1 u *} + e^{-is_1 u *} + e^{is_2 u *} + \text{etc.}),$$

where $s_1 = 2\pi/v$, $s_2 = 4\pi/v$, etc. The meaning of the asterisk is now that every term is subject to the limitations above. It only appears when $m \pm s_r$ lies between (+ 1) and (− 1). We may now proceed to interpret the series in our case, which is entirely distinct from Lord Rayleigh's application.

Each term in the expansion represents a sharply bounded diffraction maximum at the back of the objective, corresponding to the effect of the infinite series of indefinitely small apertures. The first term represents the direct image. It exists only from $m = +1$ to $m = -1$. To interpret this, note that m is the measure of the angle of obliquity of the diffracted light. The phase angle is given by

$$mu_1 = (2\pi/\lambda)\, nh_1 \sin\theta,$$

n, h_1 where n is the refractive index, h_1 is the linear measure corresponding to u_1 in the
θ object plane, and θ is the angle of obliquity. To connect h and u, note that the first dark minimum ($u = \pi$) is distant from the central maximum of the illuminating disc by the interval

$$h = 0{\cdot}5\,\lambda/\mathscr{N}\!\mathscr{A}_c,$$

$\mathscr{N}\!\mathscr{A}_c$ where $\mathscr{N}\!\mathscr{A}_c$ is the numerical aperture of the condenser. We thus obtain

$$m = \frac{n\sin\theta}{\mathscr{N}\!\mathscr{A}_c} = \frac{\text{numerical aperture of diffracted beam}}{\text{numerical aperture of condenser}}.$$

Thus the existence of the term 1^* above between $m = +1$ and $m = -1$ simply means that it extends over that numerical aperture of the objective which is filled by the condenser. The other diffraction maxima have the same breadth in the above sense, and the terms denote the appearance of maxima in symmetrical positions on each side of the central maximum.

For example, if

$$s_1 = 2\pi/v = 2, \quad \text{and} \quad s_2 = 4, \quad \text{and so on,}$$

the first term appears from $m = -1$ to $m = +1$,

second ,, $m + 2 = +1$ to $m + 2 = -1$, i.e. $m = -1$ to $m = -3$,

third ,, $m - 2 = +1$ to $m - 2 = -1$, i.e. $m = +1$ to $m = +3$, and so on, the maxima touching in the present case, which is the one that was mentioned above for a particular arrangement of the illumination; but the discontinuity is shown (in general) by the difference in the phase angles. Thus when $s_1 = 2$ and $u = \pi/2$, the phase angles $(s_1 u)$ of the first symmetrical lateral maxima are seen to be π and $(-\pi)$ respectively. This result therefore confirms that reached in the previous particular case. The amplitudes of the maxima are independent of the positions of the apertures relative to the point of maximum illumination. We must notice that the above results are strictly true only when a very great number of grating apertures is open or available. If the number is limited, the sharpness of the maxima disappears, and the secondary grating maxima have to be considered.

The above investigation relates, of course, to indefinitely narrow apertures. The expression must be integrated for finite apertures to make it of more practical significance†. If we have a series of vibrations expressed by

$$ae^{ip\theta}\,d\theta, \quad ae^{ip(\theta+\delta\theta)}\,d\theta, \text{ etc.,}$$ p

the sum is

$$ad\theta\,\{\cos p\theta + i\sin p\theta + \cos(p\theta + p\delta\theta) + i\sin(p\theta + p\delta\theta) + \text{etc.}\},$$

or

$$d\theta\,\{\Sigma\,(a\cos\Delta) + i\Sigma\,(a\sin\Delta)\},$$

where Δ stands for the phase angle. The resultant amplitude may thus be obtained by formal integration of the exponential expression. Thus Δ

$$\int_{\theta=u_1}^{\theta=u_1+U} ae^{ip\theta}\,d\theta = \left[\frac{a}{ip}e^{ip\theta}\right]_{u_1}^{u_1+U} = \frac{a\,(e^U - 1)}{ip}\,e^{ipu_1}.$$

Hence the effect of a finite width U of the apertures may be obtained by integration between $u = u_1$ and $u = (u_1 + U)$. Thus U

$$\begin{aligned}
\text{Amplitude} &= \frac{\pi}{v}\int_{u_1}^{u_1+U} du\,(1^* + e^{is_1u*} + e^{-is_1u*} + e^{is_2u*} + \text{etc.})\\
&= \frac{\pi}{v}\left[u^* + \frac{e^{is_1u}}{is_1}{*} - \frac{e^{-is_1u}}{is_1}{*} + \frac{e^{is_2u}}{is_2}{*} - \text{etc.}\right]_{u_1}^{u_1+U}\\
&= \frac{\pi}{v}\left[U^* + \frac{e^{is_1(U+u_1)} - e^{is_1u_1}}{is_1}{*} - \frac{e^{-is_1(U+u_1)} - e^{-is_1u_1}}{is_1}{*} + \text{etc.}\right]\\
&= \frac{\pi U^*}{v} + \Sigma\,\frac{\pi\,(e^{is_rU} - 1)\,e^{is_ru}}{is_rv}{*},
\end{aligned}$$

where the terms are taken for positive and negative integral values of r subject to the same limitation that $(m \pm s_r)$ lies between $(+1)$ and (-1). If s_r be replaced by $2\pi r/v$, the expression becomes

$$= \frac{\pi U}{v} + \Sigma\,\frac{(e^{i2\pi rU/v} - 1)\,e^{i2\pi ru_1/v}}{2ir}{*}.$$

The second member becomes

$$\Sigma\left\{\sin\frac{2\pi rU}{v} + i\left(1 - \cos\frac{2\pi rU}{v}\right)\right\} e^{i2\pi ru_1/v}\Big/2r^*.$$

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† This type of two-dimensional discussion must be independent of considerations relating to the polarization of the light, etc., etc., which would be relevant in a thorough three-dimensional treatment.

This is of the well-known form $(a + ib)\,e^{i\psi}$, from which the square of the resultant amplitude due to any term is given* by $(a^2 + b^2)$. Thus for one term

$$(\text{Amplitude})^2 = \frac{\sin^2(2\pi rU/v) + 1 - 2\cos(2\pi rU/v) + \cos^2(2\pi rU/v)}{4r^2}$$

$$= \frac{1 - \cos(2\pi rU/v)}{2r^2} = \frac{\sin^2(\pi rU/v)}{r^2}.$$

The amplitude of the rth maximum is written $r^{-1}\sin(\pi rU/v)$ while that of the central one is $\pi U/v$. This is the same proportion as that found by elementary theory† for the case of plane waves diffracted by a grating; although it seems very surprising at first sight that even when an elementary source of light is focussed in the grating the proportional amplitude of the central maximum ‡ should depend simply on the ratio of the width of the grating aperture to the width of the aperture plus bar, especially when the distribution of light in the region of the focus is observed; the difficulty only arises from our ingrained habit of thinking in terms of point foci, etc., and forgetting that disturbances actually exist far outside the region of easily visible light.

The appearance of the central diffraction maximum within the limits of numerical aperture set by the condenser, and the sharply-bounded lateral maxima at angles of obliquity depending solely on the aperture of the condenser, the grating space, and the wave-length of the light, has now been established, within the theoretical limits of the discussion, as a consequence of the wave-theory. The geometrical resolving power of the grating has been shown to be independent of the apparent concentration of light, whether one or many elements are visibly illuminated.

The theoretical discussion assumed (i) uniform radiation in all directions from the elementary apertures; (ii) an unlimited number of apertures in the object plane; (iii) an objective which is free from aberration for the back focal surface. In practice, none of these assumptions will be strictly valid, but the effect of the falling off of the radiation with increasing obliquity will only modify the results of the theory for large angles, and in the majority of cases the number of elements in regular structures will be fairly large, so that the sharpness of the boundaries of the maxima will not be seriously affected. The question of the location of the diffraction maxima is discussed below. Actually, these maxima are formed by an objective which, being of finite aperture, can focus only a certain proportion of the whole pattern. The number of maxima thus taking part in the formation of the final image will thus be definitely limited.

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[*] Schuster, *Theory of Optics*, 2nd ed. p. 17.

[†] Schuster, *loc. cit.* p. 121.

[‡] It must be remembered that the *actual* light in the central region of the diffraction pattern may be due to overlapping maxima. In cases where a dark "bar" obscures the illuminating disc, it may be found that the lateral maxima are in opposite phase to the central one and tend to destroy the central light, that is of course from the analytical standpoint.

§6. CASE OF GRATING WITH ILLUMINANT OF FINITE AREA

So far we have assumed a point-source of illumination but in practice the grating will be illuminated in critical illumination, not by one elementary image, but by many such images. All these, however, will produce series of coincident maxima, since the distribution and relative intensity of such maxima have been shown to be independent of the situation of the grating with regard to the illumination. Hence the appearance of the sharply-bounded diffraction maxima is quite general under the conditions of our problem, in which the row of apertures is parallel to one side of the rectangular aperture of the condenser.

It is also to be noted that the aperture of the objective has been considered to be restricted to a small amount except in the direction of the row of apertures.

The removal of these restrictions and the discussion of circular apertures for condenser and objective leads to a Fourier expansion involving Bessel functions, and will not be dealt with in this paper. Moreover, the general case of illumination by a disc with its centre not in the line of apertures is more complex than the comparatively simple condition dealt with, i.e. the case of the rectangular aperture when the $u^{-1} \sin u$ amplitude-distribution holds in any line parallel to a diameter of the aperture.

There seems no room for doubt, however, that Huygens's principle can be applied to such cases as this without fear. If, in figure 1, S be regarded as the grating, the dotted wave-surface to the left could have been taken as the origin of elementary disturbances each of which would be diffracted by the whole width of the grating available. Looked at in this way, the sharp definition of the maxima is comprehensible in a manner that is not realizable when the distribution of energy in the plane of the grating is considered, because the condition of the sharp concentration of the energy, as it appears to the eye, seems to imply quite vitally different conditions of diffraction; in fact, many seem to find difficulty in realizing that diffraction can occur at all.

This discussion, however, shows the perfect consistency of Huygens's principle even in this somewhat difficult case. It may even appear that the argument is one of a rather circular nature, inasmuch as the distribution of energy in the elementary image has to be calculated from Huygens's principle. The correspondence of theory and fact in this case is, however, well known, and it appeared better to start from this point.

The Abbe theory (or "principle" as it might better be termed) has long been known as an elementary method of discussing the formation of the image of a regular structure in the microscope. If the incident illumination is imagined to be resolved into groups of plane wave-trains inclined at all angles to the object plane within the aperture limits of the condenser, then the diffraction maxima appear as points or lines in the principal focal surface of the objective. Such maxima are regarded as coherent sources giving rise to interferences of the Young type. If each group of wave-fronts produces an independent set of interferences, these agree in reproducing the image pattern in that plane only which is conjugate to the object. Lord Rayleigh's paper was largely concerned with this condition, and Conrady and others have shown how the theory accounts for the increasing geometrical correspondence between the details of object and image consequent on the increase of the aperture of the objective and the inclusion of higher-order diffraction maxima.

The difficulties encountered in the application of this theory to explain certain experimental results lead to the formulation of the "equivalence principle" which would claim that, in the case of critical illumination of a grating, the object would behave as if self-luminous, as far as the resulting image is concerned. In the light of the above work we can gain some insight into the physical mechanism.

Refer again to figure 5; regard MBN as the grating plane, and consider the amplitude at a point S not in the principal focal surface of the lens, but on a spherical reference-surface struck with centre B', the conjugate to B, where B represents the centre of the illumination disc. If it were necessary to compute the amplitude at S, it would be essential to compute the optical paths from S to all the apertures in the grating. A simple discussion based on Fermat's principle would establish path differences very nearly proportional to the distances of the elements from B for a considerable range on each side. Then it would be found that, provided that the range of the grating was small in comparison with the width of the objective, the expressions used to calculate the amplitudes at S would be almost exactly the same as those given above. Only the higher-order terms would be affected. We may also consider that the phase terms appearing in the expressions above give the phase relative to that of a disturbance derived from the centre of symmetry B of the illuminating disc, which phase is uniform in the reference surface if the objective is free from aberration. Therefore the calculated maxima, hitherto referred vaguely to the principal focal surface, are now seen to have an exact meaning with respect to the reference surface.

The discussion of the resultant effects in the final image plane is straightforward for any particular elementary image-disc, but the total effect of illumination by a finite source would have to be computed on a statistical-average basis, regard being had to the possible positions of the individual discs with reference to the grating. In the present paper we shall indicate the lines of procedure for only the two-dimensional aspect of the problem.

§7. APPROXIMATE CALCULATION OF EFFECTS IN THE FINAL IMAGE PLANE

The position of points in the final image plane relative to the point of symmetry of the illuminating disc may be given in terms of u, and since the extent of the regions of uniform phase in the reference surface is determined by the aperture of the condenser, any such region considered alone will give a distribution of light

W, h', Y, f' represented by $W^{-1} \sin W$ where $W = \pi h'Y/\lambda f'$, h' is the linear displacement corresponding to W, Y the diameter of a region of uniform phase in the reference surface, and f' the radius of the reference surface.

It is simplest to assume that the lateral maxima give the $W^{-1} \sin W$ type of distribution in inclined planes, normal to the radii from the centres of the regions of the lateral maxima and intersecting at the central reference point, see figure 13. Then since there will be a fair depth of focus effect in practical cases, we may add the amplitudes, due to the successive maxima, for points in the normal plane, if we take account of their phase differences.

The disturbance from a lateral maximum will show a phase difference in the normal plane proportional to the distance from the axis. To select a simple case, assume that $s_1 = 2\pi/v = 2$ as above, so that the maxima in the reference surface touch at their edges. The phase difference at a distance h' from the centre for the disturbance due to a first-order lateral maximum will then be

$$\delta = 2\pi h' Y/\lambda f' = 2W.$$

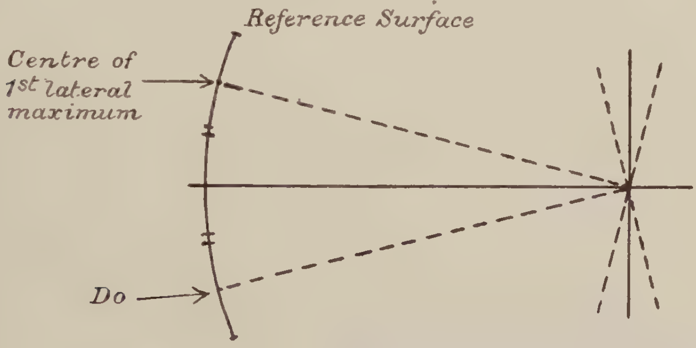

Fig. 13. To illustrate the approximate method to integrate the effects of the diffraction maxima for regions near the axis.

Hence if we go back to the elementary case where the successive maxima are represented by

$$1^* + e^{is_1u^*} + e^{-is_1u^*} + \text{etc.},$$

we may consider the central aperture of the grating to lie at the point u_1 in the illuminating disc.

The distribution of light due to these maxima in the final focal plane is therefore given by

$$W^{-1} \sin W\{1 + e^{2i(W+u_1)} + e^{-2i(W+u_1)} + \text{etc.}\},$$

and the effect of the central and diffraction maxima becomes

$$W^{-1} \sin W\{1 + 2\cos 2(W+u_1) + 2\cos 4(W+u_1) + \text{etc.}\}.$$

The symbol W represents the value of $(u - u_1)$. Hence the expression becomes

$$(u - u_1)^{-1} \sin (u - u_1)\,(1 + 2\cos 2u + 2\cos 4u + \text{etc.}),$$

which represents the contribution of a single illuminating point. It is a discontinuous distribution appearing only at $u = 0$, $u = \pi$, $u = 2\pi$, etc., and the first term outside the bracket shows that these isolated maxima will have considerable values near the region $u = u_1$ only. This is what we should expect if the objective could have an indefinitely great aperture, but the method of the discussion is not really applicable here. If, however, the aperture only includes the first two or three diffraction maxima, the distribution will not be discontinuous.

Let us, however, consider the effect of a multiplicity of illuminating points. We must integrate the intensities for all values of u_1 from $-\infty$ to $+\infty$. The intensity corresponding to the case above is

$$(u - u_1)^{-2} \sin^2 (u - u_1) (1 + 2 \cos 2u + 2 \cos 4u + \text{etc.})^2,$$

and the intensity in the image plane is therefore represented by

$$I = (1 + 2 \cos 2u + 2 \cos 4u + \text{etc.})^2 \int_{-\infty}^{+\infty} \frac{\sin^2 (u - u_1)}{(u - u_1)^2} du_1$$

$$= \pi (1 + 2 \cos 2u + 2 \cos 4u + \text{etc.})^2,$$

so that the isolated maxima now all have the same intensity. An exactly similar expression appears, no matter how many terms appear in the bracket. The effect of the illumination of a finite area of the object will simply be to remove the restriction of the $(u - u_1)^{-2}/\sin^2 (u - u_1)$ factor. If the aperture is wide enough to include only the direct image the illumination in the image plane is uniform; if the first order maxima are included the appearance consists of a series of main maxima at $u = 0$, $u = \pi$, etc., separated by secondary maxima of one-ninth the intensity of the main ones. Thus it will appear that the type of discussion, explaining the working of the Abbe principle employed by Conrady*, which was developed on the assumption of groups of plane waves diffracted by a grating and which traced the increasing similarity of image and object resulting from the inclusion of more and more of the diffraction maxima, can be applied also to the case of critical illumination.

The full discussion of the correspondence of image and object in more general conditions will be relegated to a separate paper. One or two points, however, may be emphasized.

In this way of analysing the mechanism of the image-formation a unit of the whole system would be seen by use of (i) critical illumination, (ii) an optically perfect condenser system, and (iii) a source of light so small that its image in the object plane approximates to the elementary type. Unless this illuminating disc is very small in relation to the spacing of the object details, the image cannot be regarded as other than an assembly of interferences. In practice the numerical aperture and the resolvable object detail in the microscope are found to be connected by the approximate equation

$$h = 0{\cdot}5\lambda/\mathcal{NA}, \ldots\ldots \text{(rectangular aperture form)}$$

and if the aperture of the condenser is equal to that of the objective the diameter of the illuminating disc is $\lambda/\mathcal{NA}$. The majority of cases of fine resolution (only attained by a condenser of somewhat smaller aperture) are therefore attributable to interference effects, but the application of the equivalence principle to the broad features of the imaging is fairly clear; for this is safeguarded by the appearance of the first term in the series expansion. The image is formed in part by simple convergent spherical waves of which the intensity depends simply on the transmission of the object plane around the corresponding region of illumination. The aperture of these waves is limited simply by the aperture of the illumination. Within this range the equivalence is perfect, provided that no diffraction maxima disturb the uniformity of the convergent light. We can easily see, in the light of these ideas, why the aperture of the illumination must be increased to resolve certain fine structures.

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* *Journ. R.M.S.* **163**, 610–633 (1904); **168**, 541–553 (1905).

It appears that the main advantage of critical illumination is that we can by its means most easily realize the uniform distribution of light into the objective. The interference phenomena in the field (although we still largely depend on them to secure the finest resolution) are then extremely localized. If the object is a grating the maxima are likely to have uniform intensities over their areas of the reference surface and therefore will be in the best condition to manifest interference.

The relations of the equivalence principle and the Abbe principle are now more clear. The first gives an account of the broad features of the image-formation, while the second is still a perfectly general and legitimate means of analysis of the mechanism of the image-formation, as is shown, for example, by the final form of the expression given above for the distribution of light in the image plane, when the restrictions of the illumination have been removed.

Looking at this in another way, we see that we could produce just the same effect by putting a self-luminous surface in the first principal focal plane of the condenser; then with a reasonable aperture, the light passing through the object plane can be regarded as composed of independent groups of parallel wave-trains, and we then contemplate sets of coherent homologous points in the diffraction maxima. In order to get the results in the final image plane we have to integrate the effects of all the groups of homologous points. This is a different mode of analysis, but one which must ultimately lead to a precisely similar result.

It seems necessary to point out, however, that neither the Abbe nor the equivalence principle are more than aids to the discussion of complex cases. Both have their legitimate uses. It cannot be agreed that the Abbe principle is only of interest, then, in the extreme case of illumination by indefinitely narrow pencils; it represents a mode of analysis which remains valid in all conditions, notwithstanding the difficulties occurring in its application. In both cases it is the final integration of the effects which is the real difficulty.

Credit is due to Vasco Ronchi* who pointed out the persistence of the diffraction under the conditions of critical illumination for his own observations, and drew attention to this point in connection with the theory of the microscope. In view of the practical importance of the question, it was judged advisable to examine the consistency of Huygens's principle in this connection, and also to arrange for further confirmatory experimental work. The importance of the experimental work is very considerable because all deductions based on the elementary forms of Huygens's principle are bound to be approximations. The exact calculations present problems of the greatest difficulty. It may be said, however, that the results found above are well confirmed by careful experiments which have been carried out by one of my pupils and which will be published in due course.

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* *Zeit. f. Phys.* **46**, 594 (1928).

APPENDIX

The constancy of phase for the Airy disc

With reference to figure 7, let PAQ be a horizontal diameter of a symmetrical convergent wave-front, free from aberration, and centred in the point B'. Let $B'C = h'$ be the radius of the first dark ring of the diffraction pattern, then

$$h' = \cdot 61\lambda f/Y,$$

where $AB' = f$ and $AP = Y$. If we join AC, and drop a perpendicular from B' on AC, we see that we may put, within very close approximations,

$$AC - AB' = h' \sin C\hat{A}B' = h'^2/f = \cdot 4\lambda^2 f/Y^2.$$

This will be a very small quantity in comparison with λ, even for values of h' corresponding to many interference rings. Hence the wave-disturbances arriving from the point A at points in the focal plane surrounding B will be sensibly in the same phase for a distance very large in comparison with the visual limits of the Airy disc.

Considering a disturbance arriving at C from R, figure 7, we can, by dropping a perpendicular from C on to the line RB', find that (very nearly)

$$RB' - RC = h' \sin \alpha'.$$

Hence disturbances from R will reach the point C with a lead in phase as compared with those arriving at B' from any point of the wave-front. This lead in phase will be

$$2\pi n h' \sin \alpha'/\lambda,$$

n where n is the refractive index.

On the other hand, disturbances arriving at C from the point S will have a numerically equal lag in phase as compared with those arriving at B' from any point of the wave-front, say from the point A. In practical cases the slightly different distances of R and S from the point C will not cause any noteworthy difference of amplitude in the disturbances from equal elements at R and S.

Now two disturbances having equal amplitudes but differing phases must produce a resultant having a phase which is the mean of the phases of the components. Therefore the phase of the resultant at C is the phase of the disturbance from A, which we have shown to be sensibly constant. We may note, however, that if the two phase-angles lie between 90° and 270°, then the resultant amplitude will have a negative sign as compared with the positive amplitude-contribution from A.

This discussion applies so far to the disturbances actually derived from the line through R and S. If we visualize the spherical segment of the wave-surface and $B'C$ as the polar axis of this sphere, then we can divide up the segment into strips of latitude (perpendicular to the diagram in figure 7) and realize that points such as C will be equidistant from all points on such a strip. The action will be as though the whole strip were concentrated at its mid-point, and the effect produced at C by the whole segment is reduced to that of a line source with constant phase but with amplitude diminishing from the centre outwards symmetrically on each side. The amplitude at the symmetrical points R and S being equal, we may now apply the above argument. Since any pair of symmetrical points can occasion no difference of phase in the resultant, the same consideration must apply to the effect of the whole wave-surface.

It is worth noting that a similar argument applies to many other cases in optics, such as a system of interference fringes. The phase is constant over the system, but the amplitudes of successive fringes alternate in sign.

DISCUSSION

Mr T. Smith. Explanations of distinctly varied characters have been offered to account for the effects seen with microscopes, but a mathematician will not wish for anything but a wave theory, i.e. a theory which allows the light-intensity seen at any point to be regarded as the effect due to disturbances of suitable amplitudes and phases imposed at an arbitrarily selected surface surrounding that point. The choice of surface is merely one of convenience, and it is possible to obtain very approximate results under certain conditions by neglecting the disturbances over all but a part of the surface. One of the novel features of Abbe's principle lay in selecting a different surface from that which might have been thought convenient. With certain types of object the distribution of light in this surface is markedly irregular, and its appearance affords the microscopist a guide as to the suitability of his optical system for such objects. Abbe's view, however, does not appear to me to be in general an advantageous way of regarding the action of a microscope. It is easy to understand how it came about that incorrect conclusions on the relative advantages of direct and dark-ground illumination were drawn from it, though Abbe's adherents do not seem to have realized that the inferences were wrong. The disagreement of experiment with this view is responsible, if I understand aright, for the appearance of the equivalence theory. If this theory goes so far as to claim that no difference exists between the images of self-luminous and illuminated objects, it must be rejected: if it goes less far it is useless unless it points out the nature and magnitude of the differences.

The real difficulties in constructing a theory of the microscope lie in our ignorance of what occurs when the light reaches the object. We have in consequence to consider a variety of ideal objects with assumed properties, which may be very far from those of the natural objects the microscopist wishes to examine. The characteristic peculiarity of the microscope is that large angular apertures are involved, and this makes it necessary for an adequate general theory to take account of vibrations in three dimensions at the object. With certain objects the final result at the retina of the observer may be the same or nearly the same as that which follows from a two-dimensional treatment such as that of the present paper, but in others the results may call for appreciable modification. Prof. Martin does well to emphasize the importance of amplitudes where the energy may be small. It may be recalled that on these grounds Abbe's theory seemed to the late Lord Rayleigh to require theoretical justification before the correctness of its precise doctrine was evident.

In considering the mathematical results of the paper it may be helpful to remember that two factors are involved in optical imagery, viz. a point-to-point correspondence between object- and image-space, and adequate contrast in the vicinity of the image. With some instruments the two go together almost automatically, but this is not invariably the case. The distinction is allied to that between a simple and a complex system of waves. Any investigations which help us to interpret more accurately the appearances presented by microscopic images should be most gratefully received, and most critically examined.

Mr J. RHEINBERG. I should like in the first place sincerely to congratulate the author on a fine and very useful piece of work.

There have probably been few subjects on which during a period of half a century there has been more controversy than that of image-formation in the microscope. The well-known Abbe or diffraction theory was assailed at the outset; other theories based on the work of Airy, Rayleigh and others were considered to show that it could not hold good under the ordinary working conditions of the microscope, and even as recently as 1929, when a paper by Dr H. Moore, of the British Scientific Instrument Research Association*, led to many papers and a general discussion on the subject by the Royal Microscopical Society, it was again shown how much confusion still exists, and how the various theories, or (as the author more correctly terms them) principles, are supposed to contradict one another. By mathematical treatment, the author has now shown us that there is no want of harmony, no necessary discord, and no contradiction between these principles.

As long ago as 1902 I published a paper, the title of which ("The Common Basis of the Theories of Microscopic Vision Treated Without the Aid of Mathematical Formulae")† shows that already at that time I was arriving at similar conclusions, and the abstract of my contribution to the 1929 discussion at the Royal Microscopical Society‡ starts as follows: "It is shown that, as regards the formation of the image by the microscope, the Abbe theory and the Airy theory and its variants do not in any way conflict. The Abbe theory, by reason of the way it originated, deals in the first place with objects so illuminated as to lack self-luminosity to the maximum extent. The Airy theory deals with completely self-luminous objects. These are the two limiting cases. A gradual but strictly limited approach towards self-luminosity is presented by the structural elements of objects according to the way they may be illuminated." I have never used other means than careful experiments, physical concepts and analytical methods to arrive at my conclusions; I was not aware that the author was going into the subject mathematically, and am therefore the more pleased that there should be so great a correspondence in the conclusions arrived at.

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* *J. R. Mic. Soc.* **48**, 133–143 (1928).

† *Zeit. für wissenschaftliche Mikroskopie*, **19** (1902).

‡ *J. R. Mic. Soc.* **49**, 132–143 (1929).

One or two matters arise out of the author's work to which I should like to call attention. The chief one concerns the relative applicability of the so-called "equivalence" and the Abbe principles. The "equivalence theory" is a term that has sprung up only of recent years. I believe that Dr Moore imported it from Germany from the papers of Prof. Berek of Marburg, but what it denotes has not, so far as I am aware, before now been properly defined. The author has made clear in his paper, and in the few words which he spoke in introducing his paper to the meeting, that he understands the term as meaning the equivalence between the behaviour of an illuminated object and a self-luminous one, and that is the sense in which I regard it in the following remarks. Now the concept of a self-luminous point is one which emits coherent rays in all directions, giving rise to a wave-front of regular amplitude and light-intensity; a self-luminous area consists of an aggregation of such points each preserving its own characteristics, whilst they do not bear any established phase-relations amongst themselves. The focussed image of a point in the object plane of the microscope is, however, a diffraction or Airy disc, i.e. an area consisting of points all of which are in phase-relationship with one another, besides being of variable amplitude and light intensity; and the focussed image of an illuminant of finite area consists of the overlapping of such diffraction discs. The area covered by a single diffraction-disc may cover many elements of the object itself, so that in an object of regular structure, such as a grating, many elements of the object may be emitting or passing coherent rays, instead of the non-coherent rays which would be emitted by self-luminous points similarly spaced. The author's figure 10, for example, clearly shows how the Airy disc (taken only as far as the first dark ring) may cover many object elements. To make our object as nearly self-luminous as may be possible, we must therefore contract the size of the Airy discs. This can be done only by increase of the aperture of the condenser-lens which is forming the Airy disc in the object plane, but the point to be noted is that even so we can get nowhere near the ideal of self-luminosity. Therefore, in the vast majority of cases we are going far more safely if we apply the Abbe principle than if we apply the so-called equivalence principle.

Any natural object, even if of supposedly regular structure, is so complicated and intricate in its constitution—the refraction, absorption and polarizing properties and the colour of its elements all play their part—that it simply defies adequate mathematical treatment. It has been one of the most frequent complaints against Abbe that his theory or principles were based upon experiments on artificial preparations under special conditions of illumination which are not found to be the best in practice. That was inevitable, and I would only note here that others who have tackled the problem of image-formation in the microscope, whether by experimental, analytical, or mathematical methods, including Rayleigh, Johnstone Stoney, and now the author have also found it absolutely unavoidable to adopt the same course. The working microscopist, however, cannot enter into involved mathematics, even if he has the ability; for him the necessity is to have a quick means of estimating to what degree the image of the object which he sees may be likely to correspond to the structure of the object itself, and to find out rapidly whether that image can be improved. Herein, to my mind, lies the supreme value of the Abbe principles, for the laws established from these have furnished us with a reference-plane, namely the back focal plane of the objective, easily examined, in which light phenomena occur (or can readily be made to occur) which afford us the best clue to the interpretation of the image formed in the final image-plane of the object, and also tell us in many cases how to illuminate our object to the best advantage. Microscopists are familiar with the light distribution of maxima and minima occurring in this plane when the natural or the artificial object viewed is one having approximately regularly repeating elements; with care, analogous phenomena may be observed with objects consisting of only one or two elements. These phenomena are due simply to the fact that the light in the object-plane does not consist of an assemblage of points emitting non-coherent rays, as it would if this object were self-luminous; we are dealing with *images* of self-luminous points in the object plane, consisting of an assemblage of points which pass on coherent rays that can mutually interfere. Even if we have arranged for so-called "critical illumination" of our object, so that the phenomena are obscured, we only require to cut down the angle of the illuminating cone temporarily, by means of the iris of the condenser, to obtain the required information.

The equivalence principle is helpless to afford the microscopist any such experimental insight or information as to what he may be doing. So long as this principle which, as the author says, has not involved any new physical conceptions, is held to be in opposition to the Abbe principles, it just prolongs and accentuates the confusion already too pronounced as to the mode of formation of the microscopic image. If in the hands of competent physicists and mathematicians the equivalence principle proves a tool to throw light upon some isolated problems which are difficult to solve by the Abbe principles, so much the better, although personally I doubt whether it will do so. It seems to me too limited in its application in relation to the problems of the microscope. The great merit of the author's work is that by mathematical treatment he shows that there is no opposition between the so-called equivalence principles and the Abbe principles, and for my part I think he has rendered a notable service to microscopy in doing this, because it helps to dispel the doubts of those microscopists who have had such doubts—about the methods hitherto employed as their surest guide in the interpretation of the microscope image.

Dr H. MOORE. The results obtained by the author are of interest on theoretical as well as on practical grounds. In the first place they throw considerable light on the vexed question of the validity or otherwise of the Abbe theory. Briefly, Abbe pointed out that any object having a regular structure would so diffract the light that, if the aperture of the object-glass were sufficiently large, diffraction-images of the source would be formed in or near the back focal plane of the object-glass and, as a result, interference-fringes would be produced in the image-plane. Abbe's theory, as expounded by Dippel and others in statements made with Abbe's express approval, stresses the importance of these interference-effects to the extent of claiming that they are the only physical processes involved in the formation of the images of non-self-luminous objects in the microscope, irrespective of the conditions of illumination and irrespective, also, of the structure of the object. In other words, according to the Abbe theory, the ordinary processes involved in the formation of the image of a self-luminous object play no part in the formation of the image of a non-self-luminous object.

As is to be expected, the author's mathematical analysis shows clearly that the interference effects to which Abbe drew attention do arise, but it shows also that, with critical illumination, "The image is formed in part by simple convergent spherical waves of which the intensity depends simply on the transmission of the object plane at the corresponding point of illumination," i.e. by the processes involved in the production of images of self-luminous objects. This result is a direct contradiction of the Abbe theory.

From the practical point of view the results will be of special interest to those who, for any reason, may have to work under conditions in which the illumination of the object must be restricted in some way or other. Mathematical analysis of the distribution of light in the image plane, such as that given in the paper, should afford material assistance in deciding, in such cases, which of the available methods of illumination will reduce to a minimum the visibility of interference-effects arising from any regular structure in the object and which, therefore, will give rise to an image having the closest resemblance to the object. From the practical point of view, also, particular interest attaches to one of the conclusions in the present paper, viz.: "The majority of cases of fine resolution are therefore attributable to the interference effects...." This is in conflict with the views of many microscopists, provided the illumination of the object be properly arranged. It will be interesting to see the further papers which the author has promised us, in which he proposes to deal with more general conditions of illumination, and to see how far this conclusion needs modification as a result of more general treatment.

AUTHOR's reply. I am fully aware of the divergence between the complexity of practical cases and the artificial simplicity of the theory. Also it is not easy to realize what the implications of the formulae really are. An example of this is shown in my use of a misleading word in a sentence which is quoted by Dr Moore, and which I intend to change in the paper. Instead of the words "at the corresponding point of illumination," I really meant "around the corresponding region of illumination," because, as Dr Moore will agree, the transmission at a single point cannot be significant. Thus, in the case of the grating, the simple direct spherical wave of the theory has the relative intensity $\pi U/v$ which is the average transmission of the object plane. I think we ought to be very careful in applying these results to cases involving quite other conditions, and I hope that those interested in the subject will think over the question again in the light of the present work. I do not think my results contradict the older principles, but they do reveal their limitations.

SOME STUDIES IN PYROMETRY AND ON THE RADIATION PROPERTIES OF HEATED METALS

Summary of lecture delivered on October 17, 1930

By Dr RUDOLF HASE,

Dozent für Wärmemesstechnik an der Technischen Hochschule, Hannover

The investigation of the spectral energy radiated by solid and liquid metals at high temperatures is of double interest. Firstly, from a technical point of view, radiation in both the visible and the infra-red portion of the spectrum seems to be the simplest (and in many cases the only) means possible for high-temperature measurement, provided that we know the laws connecting temperature with distribution of spectral energy. Secondly the modern development of physics has proved that the study of energy emitted by matter is of increasing importance for research on atomic problems.

Ideal black-body conditions, for which the Wien-Planck relation holds true, will be met with rarely in practical pyrometry. This is a well known fact as far as measurements of iron ingots, molten metals in crucibles and the like are concerned, but it is necessary to draw attention to the fact that even the radiation of closed ovens, such as Martin furnaces, is not a totally black one. Even if corresponding measurements in different wave-lengths of the visible spectrum with two pyrometers, one having a red and the other a blue filter, give the same temperature reading, thus apparently indicating the blackness of the enclosure, difficulties arise as soon as we alter the method by measuring the total radiation. Deviations from black radiation are known to become more noticeable the farther we move towards the longer wave-lengths. In the case mentioned the deviations may obviously be ascribed to the absorption bands of vapour and carbon dioxide. The problem of an industrial furnace, uniformly heated by gas or oil, has not been solved yet. In fact there are always differences of temperature between iron bath, cover, side-walls and flames, and it is easily understood that the selective emissivity of the hot gases must have great influence, to say nothing of the absorbing cooler gases in the channel-shaped sight-holes, which attenuate the infra-red radiation but have no influence in the visible spectrum. Similar considerations hold true for the majority of industrial temperature-measurements.

This interpretation explains in a satisfactory manner the divergent results obtained by different methods, if we correct the hitherto unchallenged opinion regarding the black-body conditions in industrial furnaces. Evidently modern pyrometry is not a problem of instrument-making, the essential point being a critical understanding of the heat-exchange between the body to be measured and its surroundings.

Difficulties increase if we consider the radiation conditions of glowing metals outside the furnace in the open air. In this case the surrounding temperature is so low relative to the heated body, that we can neglect the influence of absorbed or reflected radiation and speak of the proper radiation of the metal in question, or better still of its surface radiation. Excepting in the case of transparent materials, such as molten glass, the emission is now to be attributed to the outer atomic layers of a heated solid or liquid body.

Since most of our industrial metals oxidise at higher temperatures one has to ascertain whether the radiation observed belongs to the pure metal or to its oxide. But even if we know the spectral energy curves for both substances, a further uncertainty is involved by the existence of different oxides. In the case of iron the amount can vary between a scarcely visible film of oxide and a dark layer of scale. The latter, although having a greater emissivity, forms a solid crust of bad thermal conductivity, being often separated from the metal by insulating layers of air, and so gives the effect of too low a temperature. The same may be said of liquid metals. Still, in the case of iron my own investigations* have shown that beginning at about 1350° C. the oxide layer gets more and more dissolved in the liquid iron, the latter thus with increasing temperature assuming more and more the radiation-properties of the pure metal. Below this limit, between 1200 and 1400° C., the oxidising power is so effective that the molten iron, leaving the tap-hole of the cupola in a fairly pure state with an emissivity of only 0·35, after one metre's run in contact with the open air has acquired an emissive power of nearly 1·0, i.e. that of a black body. Further details may be found in the publication mentioned.

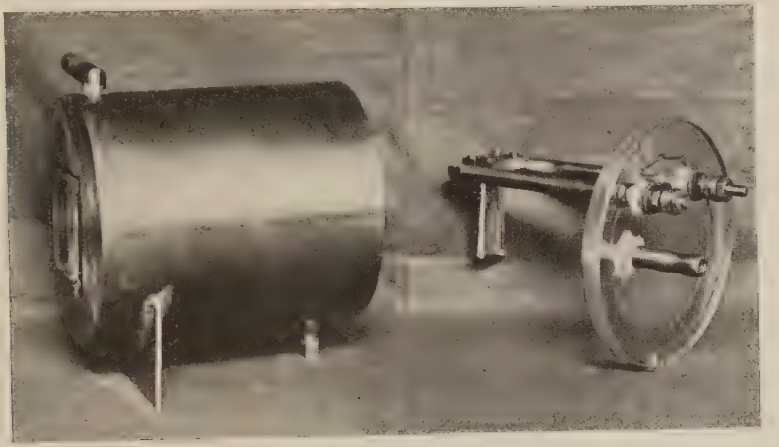

Fig. 1.

To obtain proper and well-defined experimental conditions I investigated various metals in the form of U-shaped sheets mounted on a frame, figure 1, inside a water-cooled box, which could be evacuated or filled with argon. The true temperature of the electrically-heated metal sheet was determined optically, by observation of the narrow space between the two planes†. The radiometric observation was carried out with a rocksalt spectrometer with vacuum thermocouple and string-galvanometer, through a rocksalt window sealed into an opening in the front wall of the metal box.

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* *Stahl und Eisen* (1930).

† Mendenhall, *Phys. Rev.* **33**, 74 (1911).

By Maxwell's theory, Drude and Planck found* the relation

$$R = \frac{2\sigma\tau - 2\sqrt{(\sigma\tau)} - 1}{2\sigma\tau + 2\sqrt{(\sigma\tau)} + 1},$$

where R is the reflectivity,

σ the electric conductivity

and τ the frequency of the radiation,

which leads by approximation to the equation

$$R = 1 - 2/\sqrt{(\sigma\tau)} = 1 - A.$$

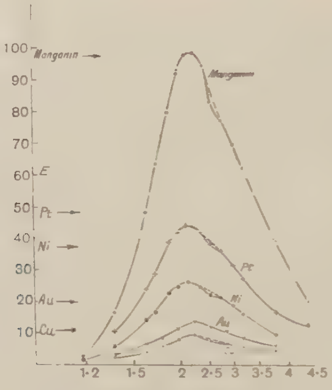

Fig. 2. $E_{\text{max.}}$ ($T=2640$).

As, according to Kirchhoff, emissivity is proportional to absorptive power, we obtain with specific resistance w and wave-length λ

$$E = 2/\sqrt{(\sigma\tau)} = \text{constant} \times \sqrt{(w/\lambda)}$$

on the assumption that we confine ourselves to the longer wave-lengths. That means that the emissivity of a pure metal is to be expected to be equal to the root of the specific electrical resistance. Corresponding observations of the metals gold, copper, nickel, platinum and manganin gave the isochromatic curves of figure 2. It will be seen that the ordinates of the energy maxima increase in the same order as the specific resistance. The true positions of the values of $\sqrt{w}$ are indicated by little arrows, in most satisfactory agreement with the data obtained, as the deviations must be ascribed to inevitable impurities of the metal surfaces. The most striking fact brought out by these observations is the validity of the Planck-Drude relation for wave-lengths down to 2μ.

Combining this equation with the Wien-Planck law for the spectral energy-distribution, we find for the intensity-maximum

$$E_{\text{max.}} = \text{constant} \times \sqrt{w_0} \, . \, T^6.$$

* P. Drude, *Physik des Aethers*, **2**, 575 (1894).

That is to say, the maximum of the energy emitted by a radiating metal is proportional to the root of the specific resistance at zero and to the sixth power of the absolute temperature, instead of to the fifth power as in case of a black body.* This sixth-power law has been stated hitherto only for platinum by Lummer and Pringsheim, but it appears to be a universal one. Figures 3 and 4 give the energy curves for iron and nickel obtained with the rocksalt spectrometer previously mentioned.

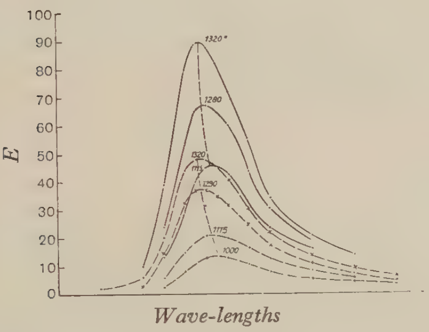

Fig. 3. Isochromes of iron:
Black body •—•—•—•—•
Clean iron in argon +−−+−−+−−+

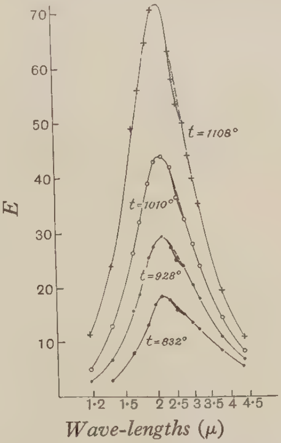

Fig. 4. Isochromes of nickel.

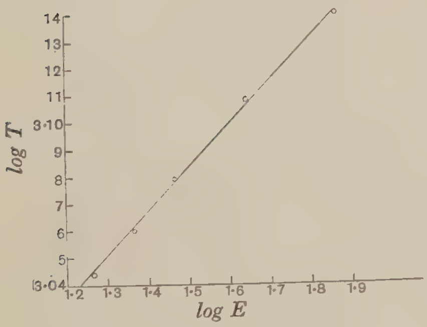

Fig. 5. Relation between log *T* and log *E* for nickel.

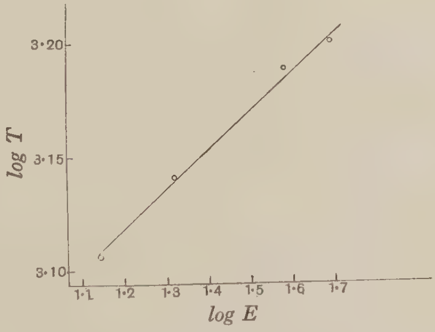

Fig. 6. Relation between log *T* and log *E* for iron.

In the case of iron, I succeeded at last in sustaining a clean surface in a current of argon for at least the duration of each curve-measurement. For comparison the black-body curves for the same temperatures are plotted. It must be added that all observations were carried out by immediate comparison with a black body, to eliminate errors due to atmospherical and optical absorption. While the iron isochromes, figure 3, show fairly well the displacement of the maximum similar to that given by Wien's law, this appearance seems not so marked in the nickel curves, figure 4. But if we make use of the data given by those curves and plot the logarithm of the absolute temperature against the logarithm of the maximal emitted energy, we shall find remarkably straight lines, figures 5 and 6, following the equations

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* Aschkinass, *Ann. der Phys.* **17**, 960 (1905).

$$\log E_{\text{max.}} = 6 \log T - 17{\cdot}5,$$

$$\log E_{\text{max.}} = 6 \log T - 17{\cdot}0$$

respectively. This means that the sixth-power law of Aschkinass is obeyed not only by platinum but also by pure iron and nickel in the temperature-range observed. Corresponding observations are being carried out on other metals.

In any case the results of the present investigation are in good agreement with the theoretical predictions, based on Maxwell's theory, which assert an interesting relation between electric resistivity and emissivity of metals.

DEMONSTRATIONS

Demonstration of plug and ring gauges given on January 16, 1931, by F. H. Rolt, National Physical Laboratory.

The object of the demonstration was to show how the degree of fit between a highly finished cylindrical plug and a ring gauge, both of hardened steel, varies with the difference between their sizes. A ring and a series of plugs of graded sizes were used for the experiments. The actual sizes of the various gauges were as follows:

Ring		1 + 0·00005 in.
Plugs	*D*	1 − 0·00010 in.
	H	1 + 0·00000 in.
	J	1 + 0·00005 in.
	F	1 + 0·00015 in.

With the surfaces of the gauges quite clean, the plugs were inserted in turn into the ring. Plug *D* was an easy fit, and with the difference of 0·00015 in. between them it was possible to spin the ring freely upon the plug when the latter was held horizontally. The next plug *H* could still be inserted freely by hand, but plug *J* (which had the same measured size as the ring) required gentle hand-pressure to fit it into the ring. The last plug *F*, which was 0·0001 in. larger than the ring, could not be inserted by hand-pressure, but the operation became possible after the surfaces of the gauges had been lubricated with vaseline. This apparent fitting of a plug into a ring smaller than itself is explained by the fact that the ring expands, assisted largely by the powerful wedging-effect brought into play by even moderate axial pressure, as the result of the usual slight taper at the advancing end of the plug. The increase in size of the ring has been verified by measurements taken on its outside diameter before and after insertion of the oversize plug.

It was further demonstrated that when the surface of the smallest plug *D* also was coated with vaseline, this plug became almost as tight a fit in the ring as the oversize plug *F*. A film of grease thus has the effect of masking entirely the presence of both positive and negative differences of a small order.

Demonstrations were given also of the adherence which can be obtained between gauges of the Johansson type, which consist of rectangular blocks of hardened steel, one opposite pair of faces of each block being finished optically flat and parallel.

A small set of these gauges ranging from 0·1000 in. to 0·1001 in. by steps of 0·0001 in. was shown. It was demonstrated that the difference between two neighbouring gauges of this set could be seen against a strong light by the wringing of the two gauges side by side on a truly flat surface and the laying of a knife-edge straight-edge across their upper surfaces. In such a test, the slit of light between the straight-edge and the smaller of two gauges can no longer be seen if the difference is less than about 0·00003 in. It was shown, however, that a difference as small as 0·00001 in. can be detected by preliminary light smudging of the bright surfaces of the gauges with a finger and subsequent scraping of the straight-edge gently across them. From the extent of the bright mark thus produced on the surfaces it was possible to say which was the larger of a pair of gauges differing by only 0·00001 in.

Demonstration of some stroboscopic effects, *given on December* 5, 1930, *by* Prof. G. B. Bryan, Royal Naval College, Greenwich.

The demonstration was intended to show the properties of a form of stroboscope developed by Prof. Bryan during the last three years. It consists simply of a gas vacuum-tube attached radially to a disc driven by the shaft whose speed is to be determined. The discharge through the tube is produced by a small induction coil with a tuning-fork interrupter. The outer electrode of the tube is earthed to the shaft and the inner one is connected to an insulated axial pin on which rests an insulated metal brush. The coil produces f almost instantaneous flashes per unit time, where f is the frequency of the fork, and when the disc is revolved a spoked pattern is formed owing to the persistence of retinal vision (about $\frac{1}{15}$ sec.), and at certain speeds this pattern is quite steady. The number of spokes in the pattern and the frequency of the fork determine the speed of the shaft. The great advantage of this form of stroboscope is that the whole brightness of the tube is made use of to form the image on the retina, and as this brightness is considerable, especially if the gas be neon, the patterns formed are easily seen in a sunlit room.

Two forms of vacuum tube were shown. One consists of an ordinary spectrum tube with a capillary about 3 in. long and having a tap at one end so that it can be pumped out *in situ*. It can be filled with any gas, but air gives very good results. This form gives a fine-line pattern, but owing to its size is not satisfactory for speeds above 1500 r.p.m.

In the second form the small neon tube from a sparking-plug tester is used. This is a thin-walled tube, about 1·5 in. long and $\frac{3}{16}$ in. in diameter, having external electrodes. It is mounted radially in a small metal box 3 or 4 in. in diameter and $\frac{3}{4}$ in. deep, with a glass front. The axial end of the tube is carried in a spring socket of ebonite and the outer end in a small socket in the rim of the box. This mounting is very like that of a pump on a bicycle and the tube is easily detachable. The ordinary ignition coil is too strong for this tube but gives excellent results if arranged with a parallel spark gap about 1 mm. long and a series gap about 0·5 mm. long. The series gap cuts out the faint discharge that occurs on make. This mounting can be run with safety at 4000 r.p.m.

The following experiments were made to illustrate the use of the stroboscope: (1) measurement of speed of a d.c. motor, (2) control of speed of motor, (3) measurement of slip of induction motor, (4) demonstration of falling into step of an auto-synchronous motor; displacement of motor as load is put on, and falling out of step when maximum load was reached, (5) deceleration of motor.

REVIEWS OF BOOKS

Radiations from Radioactive Substances. By Lord Rutherford, J. Chadwick and C. D. Ellis. Pp. xi + 588. (London: Cambridge University Press.) 25*s*.

The advent of this important work inevitably suggests comparisons with its distinguished forerunners—Rutherford's *Radioactivity* in its editions of 1904 and 1905, and the *Radioactive Substances and their Radiations* of 1912. No strict comparison is, however, possible. The two—or rather three—earlier treatises were concerned almost entirely with the progress of radioactive science under the inspiration and guidance of the transformation theory, and with its phenomenally rapid expansion into the whole range of classical atomics. By contrast, the new treatise derives its inspiration mainly from the nuclear theory, which was in its infancy when the *Radioactive Substances* went to press.

It is therefore in all essentials an entirely new work, and it supplements rather than supersedes the treatise of 1912. The properties of the α, β and γ radiations are described and discussed in the greatest detail, and with particular reference to their bearing upon problems of atomic and nuclear structure. Brief accounts are added of recent work on the penetrating radiation in the atmosphere and on the non-radioactive isotopes. As is indicated by the title, there is only the briefest description of the properties of the radio-elements; the book is, however, made largely self-contained by the insertion of concise but adequate summaries of the transformation theory and of the most recent values of the fundamental radioactive constants.

Some of the more important and more beautiful of the earlier experiments—such as the first α-particle counting—are included, but in the main the book deals with the advances of the past few years. The distribution curve of the numerous references to original papers gives no indication of having traversed a maximum—a large proportion of the references is to papers published since 1925—and even if the work had been no more than a compilation, it would clearly have been difficult of fulfilment by a single hand.

The senior author, on every occasion on which he has undertaken a comprehensive account of the current position in radioactive science, has been committed to the writing of a new treatise rather than to the revision of an existing work—a penalty mitigated only by the circumstance that it has so largely been a result of the activities of his own research schools in Montreal, Manchester and Cambridge. On this occasion he has taken into collaboration two of his colleagues, both actively engaged in research on the problems under review. Their close cooperation has resulted in a book which is singularly free from the discontinuities which are commonly associated with a composite work.

The new book will be welcomed as an authoritative statement of the present positions of the many sides of modern radioactive research. The success of its appeal to specialists in radioactivity was assured from the outset, but the book can also be recommended confidently to a wider circle of readers—namely, to all who are interested in the trend of atomic theory. Its appearance is doubly opportune, both for the specialist and for the general reader, in view of the continuous rapid expansion of the subject and of the developments which may be expected to result from the application of the new mechanics. And, in conclusion, the book will bring much encouragement to those who still persist in regarding physics as a predominantly experimental science.

The Physical Principles of the Quantum Theory. By Werner Heisenberg. Translated into English by Carl Eckart and Frank C. Hoyt. Pp. xii + 186. (London: Cambridge University Press.) 8*s*. 6*d*.

This volume is based on lectures delivered by Professor Heisenberg at the University of Chicago in the spring of 1929. The leading part which the author has taken in the development of the new quantum mechanics is recognised by all those who have attempted to follow its growth and, as Professor A. H. Compton remarks in the foreword: "the *uncertainty principle* has become a household phrase throughout our universities." Classical theories assume the possibility of observation without perturbation of the object under investigation. Modern quantum theory denies this possibility, and on this denial may be based the main argument of Heisenberg. "Every experiment destroys some of the knowledge of the system which was obtained by previous experiments."

The book contains five chapters, and an appendix in which the formal mathematical apparatus has been collected. The first chapter is of an introductory nature and contains an account of the fundamental concepts of the quantum theory. It is shown that both matter and radiation possess a remarkable duality of character, as they exhibit the properties sometimes of waves, at other times of particles. The solution of this difficulty here proposed is that the two mental pictures are both incomplete and have the validity of analogies which are accurate only in limiting cases. Mathematics is not subject to the limitations imposed by the formation of mental pictures, and it has been possible to invent a mathematical scheme which seems adequate for the treatment of atomic processes. The limitations of the concept of a particle are obtained by considering the concept of a wave, and conversely one may derive the limitations of the concept of a wave by comparison with the concept of a particle. Chapter II deals with the critique of the corpuscular theory, and chapter III with the critique of the wave theory, the uncertainty relation being made the basis of each discussion. The next chapter deals with the statistical interpretation of quantum theory, and the last chapter contains a discussion of certain important experiments. The author concludes that the progress of physics is far more likely to result in further limitations on the applicability of classical concepts than in the removal of those already discovered. In spite of its difficulties this is a book which cannot be neglected by the serious student of modern physics.

H.S.A.

(1) *Recueil d'Exposés sur les Ondes et Corpuscules*; (2) *Introduction à l'Etude de la Mécanique Ondulatoire*. By Louis de Broglie. Pp. 80. (Paris: Herman et Cie.) 29 fr.

In these two volumes Louis de Broglie gives an account of the new wave-mechanics which he did so much to initiate some six years ago. The first contains five articles which appeared between 1927 and 1929, giving a more or less popular account of the origin and development of the new theories. Whether a particular entity is to be regarded as corpuscular or undulatory in character depends far more than we had imagined on the phenomenon investigated. The same physical entity can present itself to us, according to the experiment to which it is submitted, under either the one aspect or the other. In the language of Bohr these two aspects of the entity are to be regarded as complementary rather than as contradictory. Of special interest is the last article of the series which deals with determinism and causality as affected by the physics of to-day. The author has been very successful in his aim of presenting to a wider public a general idea of the questions which are at present occupying the minds of physicists.

The second volume is more weighty and contains a detailed account of the methods of the new mechanics. Its importance is sufficiently shown by the fact that it has been translated into English by T. H. Flint and published by Methuen (1930). The French edition contains a portrait of the author and some striking reproductions, due to M. Ponte, of diffraction rings obtained by the passage of streams of electrons through thin films of zinc oxide.

H.S.A.

Conférences d'Actualités Scientifiques et Industrielles, Année 1929. Pp. viii + 270. (Paris: Herman et Cie.) 35 fr.

This most interesting volume contains nine lectures delivered in 1929 under the auspices of the Conservatoire Nationale des Arts et Métiers. These lectures were arranged to provide an outline of some of the important problems now attracting the attention of physicists, and the lecturers chosen have been very successful in presenting the philosophical principles of their subjects without entering into minute details or experiments, or into elaborate mathematical calculations. L. de Broglie explains how he was led to associate waves and quanta and he describes the rise of wave-mechanics. Foex gives an account of "les substances mésomorphes," a term including the liquid crystals discovered by Lehmann about 1890 and denoting a state distinct from and intermediate between the crystalline and amorphous states (Friedel). The magnetic properties of such substances are described and are explained with the aid of Langevin's theory. E. Bloch gives an exceptionally clear account of the theory of quanta, leading up to the most recent developments in connection with the wave properties of the electron and the principle of indeterminacy of Heisenberg. Dunoyer describes the construction and properties of photoelectric cells and gives an account of recent applications of such cells which illustrates in the most striking way how research in pure science can and does lead to most unexpected practical results. G. Ribaud supplies a lecture on the radiation from incandescent bodies and shows not only how the radiation from an ideal "black body" can be realised experimentally, but also how the measurement of radiation can be utilised in optical pyrometry. An article of exceptional interest on the electric production of sound is contributed by Colonel Jullien, who assembles a mass of information not available elsewhere with regard to the production of "electric music." Those who are not familiar with the subject will be surprised at the number of different arrangements and possibilities in this field. L. Bloch discusses the relation between the structure of spectra and the structure of atoms, emphasising the point that according to the new mechanics an atom can be revealed to us only by its radiations, so that apart from the frequency and intensity of the rays which it emits an atom has no existence for us. There is not merely correspondence, there may be said to be identity between the structure of an atom and the structure of its spectrum (Heisenberg). A more technical subject, that of high vapour-pressures and their applications in industry, is dealt with in an authoritative way by V. Kammerer. Finally M. R. Mesny discusses directed electromagnetic waves and their applications, as in the beam system of Marconi. He mentions the interesting fact that communication has been established over a distance of 8 km. with a wave-length of only 17 cm. The volume as a whole is to be recommended as providing a stimulating series of lectures on subjects of interest alike to the physicist and to the scientific layman.

H.S.A.

Modern Physics, A General Survey of its Principles. By THEODOR WULF. Translated from the German by C. J. SMITH, Ph.D., A.R.C.S., M.Sc., with 202 diagrams. Pp. xi + 469. (London: Methuen and Co.) 35*s.*

The author informs his readers that this book has been written for "non-technical experts" who, without wishing to become physicists, desire an insight into the realm of physics. The aim is to present not mere facts but their inter-relationships, and to unfold a clear view of the structure of matter. The philosophical aspect is to be emphasised, the book being indeed based upon lectures given by the author to students of philosophy over a period of twenty years.

It is divided into four parts: Part 1, the Material World; Part 2, the Atomic Structure of Matter; Part 3, the Structure of the Atom; and Part 4, the Physics of the Ether. The range traversed is very wide, and some account will be found of supra-conductivity, the Compton, Zeeman and Stark effects, electron waves and wave-mechanics, and many other subjects of comparatively recent investigation; as well as of the traditional phenomena usually treated under physics.

The aim of the author is certainly an ambitious one, for this enormous range is offered for the comprehension of the student without special equipment or training. Therein must lie our chief criticism of the book. It is not surprising to find that at many places space is taken up by a quite elementary detailed treatment of ordinary subject-matter, as, for instance, in the proof that the focal length of a spherical mirror is half the radius, or in the laborious treatment of the laws of electromagnetic induction; and that, on the other hand, in the treatment of the more abstruse parts, the stages of exposition of the theme are often glided over, as, for instance, in the theory of metallic conduction, where the failure of the simple electron theory is pointed out, and a merely nominal reference is given to the work of Pauli and Sommerfeld.

The choice of subject-matter is, for the same general reason, arbitrary in its inclusions and omissions. Many interesting descriptions occur in the course of the volume: e.g. of supra-conductivity, of the variation of specific heat with temperature, and of the distribution of elements through the earth's crust. As instances of omission, on the other hand, may be cited the facts that on the subject of the value of the velocity of sound in water no work later than that of Colladon and Sturm is mentioned; that in the section on X-rays there is no description of the spectrometer, nor indeed is the work of the Braggs mentioned; and that in regard to the dispersion of light no account is given of the electron theory—indeed the inference might be drawn that no such theory of dispersion had been developed.

Serious errors of statement are not infrequent: for example, that characteristic X-rays are excited by incident rays of the same frequency, that the Peltier effect is proportional to the absolute temperature, and that nitrogen is a paramagnetic body.

British physicists on the whole receive scant notice; no reference will be found to the work of Rayleigh, O. W. Richardson, Bragg, Lodge, Townsend, or Fleming, while the name of Sir J. J. Thomson occurs once, though Boyle is mentioned three times and Maxwell nine times.

Despite the criticism here offered it seems, nevertheless, quite likely that the book will prove of decided interest and stimulus to the discriminating reader with a fair basic knowledge of physics at his command. The translation is ably carried out.

D.O.

Manual of Meteorology. Vol. III. *The Physical Processes of Weather.* By Sir NAPIER SHAW, LL.D., Sc.D., F.R.S., with the assistance of ELAINE AUSTIN, M.A. Pp. xxviii + 446. (London: Cambridge University Press.) 36*s.*

It is now eleven years since the first part of this manual appeared as a volume of 160 pages in paper covers. It dealt with the relation of the wind to the distribution of atmospheric pressure in a way which indicated that the manual when complete would probably be for some time the standard work on meteorology. The subsequent appearance of vol. I, containing a historical introduction and general account of meteorological problems, and vol. II devoted to a survey of the globe and its atmosphere, strengthened this belief, and vol. III establishes it as a fact. The new volume treats of gravitational, sound and light waves and their effects in determining meteorological phenomena in considerable detail. Greater use is made of entropy than has been customary in meteorological works, and the authors show how valuable the idea is. The frequent appearance in the thermodynamic formulae of *tt* in place of the usual symbol *T* for absolute temperature is likely, however, to lead to misunderstandings. Quotations from original sources and in their original languages are introduced very freely, and the style is more ambulatory than in the earlier volumes.

Vol. IV, which is promised for this winter, is to deal with the applications of dynamics to meteorology.

C.H.L.

Four-Place Tables of Logarithms and Trigonometric Functions. Unabridged edition. By E. V. HUNTINGTON. Pp. 33. (London: Allen and Unwin, Ltd.) 3*s.* 6*d.* cloth, 2*s.* paper.

This is a book of four-place tables as convenient and comprehensive as any that the reviewer has handled. Besides the ordinary tables of logarithms and of logarithmic and natural circular functions, the work contains conversion tables, decimal equivalents of common fractions and of sexagesimal parts of a degree, tables of squares and cubes, square roots and cube roots, reciprocals, areas of circles, logarithms of radian measure, exponentials, Napierian logarithms, tables of radian measure, and a selected table of useful constants.

Although the present reviewer has a prejudice for eighteenth-century numerals in books of tables, he has to confess that the generous spacing of the type of uniform height employed in printing this book makes its use very easy and pleasant. Printing and production leave nothing to be desired, and the book is remarkably good value for the money.

A.F.

Les Statistiques Quantiques et leurs Applications. By Professor LÉON BRILLOUIN. 2 volumes, pp. 404. (Les Presses Universitaires de France, 1930.) 125 fr.

The advent of the Bose-Einstein statistics and the Fermi-Dirac statistics and their close relation to wave-mechanics had recently given to the subject of statistical mechanics a much more important place in physical theories than it previously held. One of the most important applications which have been made is to the theory of the electrical and magnetic properties of metals by Sommerfeld, Bloch and others, and those who are anxious to keep abreast of these modern theories cannot ignore the developments of statistical theory.

The treatise by Professor Brillouin gives an adequate account of the foundations of statistical theory and of recent applications without a wide knowledge of mathematics. Assuming an elementary knowledge of thermodynamics, he outlines the attempts to deduce by classical methods the distribution of energy in a radiative field and shows the inadequacy of classical ideas. The Planck formula is deduced not by the original method of Planck but by the recent statistical method of Bose, in which radiation is definitely regarded as made up of photons with quantised energies. He considers in some detail the two theories of light and their relation to one another, but lays considerable emphasis on the corpuscular or photon conception of light. He shows how the quantum conditions give to photons properties similar to those of waves, and how the law of diffraction of light by a plane grating can be deduced by this method (after Duane). It is interesting to note at what point the concept of photons and the quantum conditions permit an interpretation of the physical laws of optics and approach the wave point of view. The essential assumptions involved in the Bose-Einstein statistics are clearly stated and compared with those of the Fermi-Dirac statistics, the analogue of the Pauli exclusion principle in statistical theory. Heisenberg's indeterminary principle is also described and its significance in statistical applications of the new mechanics is pointed out.

The second volume deals with some of the recent applications of the new statistics. The electron theory of metals as developed by Sommerfeld and Bloch is described at some length, but the author does more than merely reproduce the works of these authors. By simple illustrations he clothes the bare bones of mathematical theory in a presentable garment of physical theory. The distribution of electrons in atoms as indicated by statistical theory is given by the method of Thomas and Fermi, and the book ends with a long discussion of the problems of ionisation of atoms at high temperatures, such as occur in astrophysical theory.

The book is well written and aptly illustrated and can be confidently recommended to those who want a simple but sound introduction to statistical mechanics.

Radioactivity and Radioactive Substances. By J. CHADWICK, M.Sc., Ph.D. Pp. xii + 116. (London: Sir Isaac Pitman and Sons.) *2s. 6d.*

There is no need to dilate on the good qualities of Dr Chadwick's little volume. It has stood the test of ten years' use and now passes into a third edition. In this edition "the text has been in some places corrected, in others amplified, so as to bring it into agreement with the later developments. The numerical data and tables have been revised." The book, which possessed original features in that it cut quite away from the historical order of development, and assumed at the outset modern notions of the structure of the atom, still retains these features. It is proverbially dangerous to generalise, but it can safely be said that the book is the best introductory treatise of its size extant.

A Text-Book on Spherical Astronomy. By W. M. SMART, M.A., D.Sc. Pp. xi + 414. (Cambridge University Press.) *21s.*

The recent growth of stellar astronomy during the last half-century has had the effect of largely attracting the attention of students away from the classical methods of the older "Spherical" Astronomy, which is concerned fundamentally with the geometry of celestial bodies rather than with their physical state. The result has been that, while a large number of treatises which bring modern astrophysics within the range of the non-technical or semi-technical reader is now available, the classical works of Chauvenet, Godfray and Ball, designed for the student of mathematical astronomy, have had few or no successors.

Nevertheless this branch of astronomy remains as fundamental as ever it was; for example, all the recent indirect methods of determining the distances of stars, clusters and nebulae, such as spectroscopic parallaxes, Cepheid periodicity, mass-luminosity law, etc., derive ultimately their justification from the trigonometric or dynamical parallaxes obtained by the methods of spherical astronomy.

On the other hand, far-reaching developments and improvements have taken place in this branch of astronomy since the above treatises were written. To name only three, we have (i) the displacement of visual by photographic observations, involving a whole new technique with elaborate mathematical machinery, in which connection, incidentally, the methods of projective geometry might often be applied with advantage; (ii) the determination of the orbits of visual, spectroscopic and occulting binaries, a subject little noticed in the older books; (iii) the calculation, which again was usually omitted, of heliographic coordinates for physical observations of the sun.

Methods of numerical computation, also, have altered and are altering. With the growing use of calculating-machines of the Brunsviga type, the importance which used to be attached to formulae "adapted to logarithms" is rapidly disappearing.

Certain classical observations, such as the determination of the solar parallax by the transit of Venus, which bulked largely in the older treatises, are now of only historical interest.

For all the above reasons, Dr Smart's new work, which brings, so to speak, the student's spherical astronomy up to date, is to be warmly welcomed. It will prove invaluable to all those who desire to make any serious study of astronomy; and it is to be noted that there is a growing tendency in universities to recognise the importance and educational value of this subject. In this, as in many other matters, London has given a lead; for many years now, it has granted a B.Sc. degree to astronomical specialists, and that this is being taken advantage of is evidenced by the many questions from London examination papers to be found among the examples in the present book.

In addition to what may be termed the classical chapters on spherical astronomy, in which the changes are mostly of arrangement and order of exposition, Chapter VII deals partly with heliographic coordinates, Chapter XI contains a very full discussion of the proper motions of the stars, including the determination of the solar motion and statistical parallaxes, Chapter XII gives a very clear account of the theory of astronomical photography and Chapter XIV of binary star orbits. Chapter XIII, on the determination of position at sea, is a very interesting reminder of the fact that astronomy still has important applications to practical life and has special value in view of the author's practical experience of navigation with the Grand Fleet during the War.

The absence of certain matters may be noticed. Thus, no attempt is made to deal with the problem of orbits of occulting variables, which receives a mere mention. Again, the book does not venture outside the old Euclidean lines and neither the three famous Einstein effects nor the influence of the curvature of the universe on observations of very distant objects are discussed. Probably the relativity theory is beyond the real grasp of a student at this stage and the omission in this case is a wise one. The problem of the occulting variable, on the other hand, is of great interest and might, one would have thought, have been presented in a fairly simple manner. Perhaps Dr Smart may consider including it in a later edition, which it is to be hoped will come soon.

The notation of the International Astronomical Union has been followed, fortunately not slavishly, and we congratulate Dr Smart on having had the courage to part company with the Nautical Almanac and introduce the term "Greenwich mean astronomical time" for the hour angle of the mean sun at Greenwich, a nomenclature that to some extent obviates the confusion introduced by a convention which no longer measures every kind of time by the hour angle of the indicating body or point.

One almost wishes, indeed, that Dr Smart had found it in his heart to disregard another recent innovation also, to wit, the reversal of sign of the equation of time, a reversal which has broken the hitherto universal rule (a rule which was a most useful memory-aid for the student) that corrections or "equations" in astronomy are always to be added to a datum of observation in order to obtain a theoretical quantity.

The examples at the end of each chapter have been carefully selected and provide an indispensable adjunct for the proper appreciation of the principles. Of the printing and diagrams it will be sufficient to say that they are fully up to the usual high standard of the Cambridge University Press.

L.N.G.F.

Photoelectric Cells. By Dr N. R. CAMPBELL and DOROTHY RITCHIE. Pp. vii + 209. (London: Sir Isaac Pitman and Sons, Ltd.) 15*s*.

As the authors remark in their preface, the two devices which make use of the effect of light in producing or varying electric currents have had singularly different histories. The selenium "cell" has been overworked by the inventor, neglected by the theorist; while the cell based on the photoelectric effect, an effect of fundamental importance in modern physical theory, has but lately attracted the attention of the inventor, and many of its practical possibilities remain yet unexplored.

The book before us is written in a spirit which realises these possibilities and realises too that a growing audience exists which is clamorous for detailed information concerning the properties and uses of photoelectric cells. That Dr Campbell is associated with the book is sufficient guarantee that the work will be clear, thorough, and, should occasion demand it, emphatic. The book is certainly remarkable in the manner in which it conveys detailed and advanced information with the assistance of a minimum of mathematics. The authors indeed 'swither,' as the Scots say, between an exposition suitable to the professional physicist and one likely to appeal to the enquirer, earnest enough in all conscience, whose mathematical equipment is limited to the proverbial four books of Euclid and algebra up to (and including) quadratic equations. This is, perhaps, in the nature of the case, inasmuch as photoelectric cells and circuits are likely to be used by all sorts and conditions of men.

The book is divided into three sections which deal respectively with the theory, use and applications of photoelectric cells. The first section discusses the photoelectric effect, photoelectric emission, the electric discharge, the voltage current characteristic and intermittent currents. The second section opens with an abstract but none the less valuable discussion of some general principles used in the measurement of physical quantities, and then proceeds to the consideration of electrostatic methods of measurement and of valve amplification. The third section is concerned with the applications of photoelectric cells, and it is not easy to find a consistent set of headings under which the many uses can be conveniently grouped. The authors, on consideration, find that the "thing studied or used, and the ultimate cause measured, compared, or detected" may be classified under one of four headings, namely, luminous flux, illumination, colour and absorption, and to each of these headings a chapter is devoted.

The volume is a storehouse of ordered information on an important and growing subject; it will, we hope, attain the distinction, rare among scientific books, of inclusion in the class of best-sellers.

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